# EXAMPLES OF SECTION 6.9

**Example 1.** *Find a general solution to the following differential equation.*

$$t^2y'' + 2ty' - 2y = 0$$

*given that* $y_1(t) = t$ *is a solution.*

**Solution.** We look for solutions of the form $y_2(t) = v(t)y_1(t) = tv(t)$. Plugging $y_2$ into the differential equation yields

$$t^2[2v' + tv''] + 2t[v + tv'] - 2[tv] = 0. \tag{1}$$

Here we have used the following fact

$$(uv)'' = u'' + 2u'v' + v''.$$

Now because $y_1$ is a solution to the original differential equation, we just ignore all the terms in (1) containing $v$. This gives

$$t^3v'' + 4t^2v' = 0. \tag{2}$$

Let $w = v'$. (2) becomes

$$t^3w' + 4t^2w = 0.$$

This is a separation of variables kind of equation. Solving it, we obtain

$$w = c_2t^{-4}.$$

Integrating this results gives

$$v = c_2t^{-3} + c_1,$$

which yields

$$y_2(t) = c_1t + c_2t^{-2}.$$

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