1. A

This is an exact equation $(2xy + \cos y)\,dx + (x^2 - x\sin y - 2)\,dy = 0$

$\Phi = \int (2xy + \cos y)\,dx = x^2 y + x\cos y + h(y)$

$\frac{\partial}{\partial y}\Phi = 2xy - x\sin y - h'(y) = 2xy - x\sin y - 2 \Rightarrow h(y) = 2y$

2. C

$y' + \frac{1}{x} y = \frac{1}{x} e^{5x}$ 1st order Linear DE with $I(x) = x$.

3. D

$xV = y \Rightarrow \frac{dy}{dx} = V + xV' = \sin(v)$

4. E

A trial solution is $(A+Bt)e^t$. Plugging into the DE $\Rightarrow y_p = (t-1)e^t$

So $y = y_p + C_1 \cos t + C_2 \sin t$. Using the initial condition, we find

$y = (t-1)e^t$

5. B

$V = 20$ $A(0) = 100$ $r_1 = r_2 = 2$ $C_1 = 10$

$A' + \frac{1}{10}A = 20 \Rightarrow A = 200 - 100\, e^{-\frac{1}{10}t}$

6. D

$\begin{bmatrix} 1 & 1 & 0 & 3 \\ 1 & 1 & 0 & 2 \\ 1 & 0 & 1 & 1 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 0 & 0 & -1 \\ 0 & -1 & 1 & -2 \end{bmatrix} \sim \begin{bmatrix} 1 & 1 & 0 & 3 \\ 0 & 1 & -1 & 2 \\ 0 & 0 & 0 & 1 \end{bmatrix}$ 3 leading 1's

7. C

$\begin{bmatrix} 1 & 0 & 2 & -1 \\ 2 & -1 & 5 & -2 \\ 0 & 1 & 1 & 0 \end{bmatrix} \sim \begin{bmatrix} 1 & 0 & 2 & -1 \\ 0 & 1 & -1 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ $X = s\begin{bmatrix} -2 \\ 1 \\ 1 \\ 0 \end{bmatrix} + t\begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

8. A

$T(1,2,3) = T(1,0,0) + 2T(0,1,0) + 3T(0,0,1)$

9. B

$$\begin{vmatrix} 2 & 1 & 0 \\ -k & 2 & 1 \\ 0 & 2 & -k \end{vmatrix} = -(k+2)^2$$

10. A

These two vector cannot be co-linear.

11. E

The number of vectors > 3.

12. E

We use adjoint to compute b_{23}. $|A| = 6$. $C_{32} = -4$.

$$b_{23} = C_{32} / |A| = -\frac{2}{3}$$

13. C

$$\left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 2 & 4 & 5 & 1 \\ 1 & -1 & k^2 & -k \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 3 & 1 \\ 0 & 2 & -1 & -1 \\ 0 & 0 & k^2-4 & -k-2 \end{array}\right]$$

When $k^2-4=0$, $-k-2 \neq 0$, this system is inconsistent $\Rightarrow$ $k=2$

4. C

The general solution to $y''+y'-2y=0$ is $c_1 e^x + c_2 e^{-2x}$.

5. B

The general solution is $y = c_1 e^{-x} + c_2 e^{2x}$. Using the initial condition, we derive $c_1 = 1$ $c_2 = 0$

6. A.

Plug $y = x^r$ into the DE $\Rightarrow$ $r(r-1) + 5r + 4 = 0 = (r+2)^2$

17. E

$(D^4 + D^2 + 1)y = 0 \Rightarrow (D^2+1)^2 y = 0$

18. D

$(D^3 - 4D)y = 0 \Rightarrow D(D-2)(D+2)y = 0 = P(D)y$

The annilator for $3\cos t$ is $A_1(D) = D^2+1 \Rightarrow y_{T.1} = A\cos t + B\sin t$

------------ e^{-2t} is $A_2(D) = D+2$ ∵ It appears in $P(D)$

So $y_{T.2} = t(Ce^{-2t}) = Cte^{-2t}$

19. E

We use V.O.P Let $y_1 = e^{-x}$ $y_2 = xe^{-x}$ $W(y_1, y_2)(x) = e^{-2x}$

$u_1' = \begin{vmatrix} 0 & xe^{-x} \\ \frac{1}{x}e^{-x} & (1-x)e^{-x} \end{vmatrix} e^{2x} = -1 \Rightarrow u_1 = -x$

Similarly, we get $u_2 = \ln x$.

The general solution is

$$y = c_1 e^{x} + \underbrace{c_2 xe^{-x} - xe^{-x}}_{C_3 xe^{-x}} + \underbrace{xe^{-x}\ln x}_{\text{particular solution}}$$

20. C

This graph implies that this system is underdamping, i.e.,

$\alpha \neq 0$ $\alpha^2 - 4 < 0 \Rightarrow \alpha \in (-2, 0)$ or $\alpha \in (0, 2)$

21. A

$$p(\lambda) = \begin{vmatrix} 2-\lambda & 0 & 0 \\ 1 & 3-\lambda & 4 \\ 0 & -1 & -1-\lambda \end{vmatrix} = (2-\lambda)\begin{vmatrix} 3-\lambda & 4 \\ -1 & -1-\lambda \end{vmatrix} = -(\lambda-2)(\lambda-1)^2.$$

22. A

$$A = \begin{bmatrix} 1 & 2 \\ 4 & -3 \end{bmatrix} \Rightarrow p(\lambda) = (\lambda - 5)(\lambda + 1)$$

$\lambda_1 = 5$. $\quad A - 5I = \begin{bmatrix} -4 & 2 \\ 4 & -2 \end{bmatrix} \quad v_1 = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$

$\lambda_2 = -1$ $\quad A + 2I = \begin{bmatrix} 2 & 2 \\ 4 & 4 \end{bmatrix} \quad v_2 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$

$$X(t) = c_1 e^{5t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + c_2 e^{-t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$$

23. A

$$A = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix} \Rightarrow \lambda_{1,2} = \pm i \quad v_{1,2} = \begin{bmatrix} 1 \\ \pm i \end{bmatrix}$$

$$X(t) = c_1 \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix} + c_2 \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} \quad X(0) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$$

$X_2(t) = -\sin t + \cos t$

24. B

We first compute the solutions to $X' = AX$. They are given by

$$X_c(t) = c_1 e^{-2t} \begin{bmatrix} \cos t \\ -\cos t + \sin t \end{bmatrix} + c_2 e^{-2t} \begin{bmatrix} \sin t \\ -\sin t - \cos t \end{bmatrix}$$

We already know the answer is B.

25. C.

A. $Ax = \vec{b}$ may not have solutions.

B $A\vec{x} = 0$ has infinitely many solutions

D. We only know A has an eigenvalue, 0.

E A is not invertible