1. The general solution of the following differential equation

$$4x^3e^{-y}dx = (x^4 + 2)dy$$

is

A. $y = C - 2\ln(x^4 + 2)$

B. $x^4 = Ce^y - 2$

C. $\ln(x^4 + 2) = Ce^y$

D. $\ln(x^4 + 2e^y) = C$

E. $\ln(x^4 + 2) = e^y + C$

2. Suppose that $y = y(x)$ is a solution of

$$(4x + y)dx + (x + e^{-y})dy = 0, \quad y(0) = 0.$$

Then y satisfies

A. $5x^2 = xe^y + xy = 0$

B. $(x + e^y)(4x + y) = 0$

C. $2x^2 + xy - ye^{-y} = 0$

D. $(4x + y)^2 + (x + e^{-y})^2 = 1$

E. $2x^2 + xy - e^{-y} = -1$

3. A tank contains 200 liters of liquid. Initially, the tank contains pure water. At time $t = 0$, brine containing 3 g/L of salt begins to pour into the tank at a rate of 2 L/min, and the well stirred mixture is allowed to drain away at the same rate. How many minutes must elapse before there are 100 grams of salt in the tank?

A. $100 \ln \frac{6}{5}$

B. $600 - 600e^{-1}$

C. $600 - e^{-1}$

D. $600 + 600e$

E. $-100 \ln(400)$

4. The rank of the matrix

$$A = \begin{bmatrix} 1 & 5 & 7 \\ 3 & 1 & 0 \\ -1 & 5 & 8 \\ 2 & 4 & 5 \end{bmatrix}$$

is

A. 0

B. 1

C. 2

D. 3

E. 4

5. If $A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then the **sum** of the entries in the third row of A^{-1} is

A. -2

B. -1

C. 0

D. 1

E. 2

6. The general solution to $ty' - y = t^2 e^{-t}$ is

A. $y = -e^{-t} + C$

B. $y = -te - t$

C. $y = -Ce^{-t} + t$

D. $y = -te^{-t} + Ct$

E. $y = te^{-t} + t$

7. Let A be an invertible matrix with the inverse

$$A^{-1} = \begin{bmatrix} 1 & 2 \\ 2 & 3 \end{bmatrix}.$$

Which of the following statement is NOT always true?

(i) For arbitrary 2×2 matrices B and C. If $AB = AC$, then $B = C$.

(ii) A^T is invertible.

(iii) For arbitrary 2×2 matrices B and C. If $BA = CA$, then $B = C$.

(iv) rank(A)=2

(v) A is symmetric.

A. (i) and (iii)

B. (ii) and (v)

C. (i), (ii), (iii) and (v)

D. (i), (iii) and (v)

E. None of the above.

8. For two $n \times n$ matrices A and B, how many of the following statements are true.

(a) $\det(AB) = \det(A)\det(B)$

(b) $\det(A) = \det(A^T)$

(c) For $k \neq 0$, $\det(kA) = k\det(A)$.

(d) $\det(A^{-1}) = \det(A)^{-1}$

(e) If $A = PBP^{-1}$ for an invertible matrix P, then $\det(A) = \det(B)$.

A. 1

B. 2

C. 3

D. 4

E. 5

9. For what α, the system of linear equations

$$\begin{cases} 2x + 5y + (3\alpha)z + 4w & = 0 \\ (\alpha - 1)y + 4z - 3w & = 0 \\ 2z + w & = 0 \\ (\alpha)z + 4w & = 0 \end{cases}$$

has non-trivial solutions?

A. $\alpha = 0, 2$

B. $\alpha = 1, 5$

C. $\alpha = -1, -5$

D. $\alpha = 1, 8$

E. $\alpha = 0, 1$

10. If $y = y(x)$ is the solution to

$$y' = \frac{3y^2 + x^2}{2xy}, \quad y(1) = 1,$$

then $y(2) = ?$

A. $-2\sqrt{3}$

B. 1

C. $2\sqrt{2}$

D. $2\sqrt{3}$

E. 0

11. Let

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & 1 & 2 \\ 2 & 1 & 1 \end{bmatrix}, \quad X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

and

$$AX = \begin{bmatrix} -1 \\ 2 \\ -4 \end{bmatrix}$$

What is x_2?

A. 11

B. 20/3

C. 5/4

D. 1

E. 0