# EXAMPLES OF SECTIONS 7.2

**Question 1.** Find the general for the system

$$\vec{x}' = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix} \vec{x}.$$

**SOLUTIONS.**

**1.** Let

$$A = \begin{bmatrix} 4 & 1 \\ -2 & 1 \end{bmatrix}.$$

First compute

$$\det(A - \lambda I_2) = \det \begin{bmatrix} 4-\lambda & 1 \\ -2 & 1-\lambda \end{bmatrix} = \lambda^2 - 5\lambda + 6 = (\lambda - 3)(\lambda - 2).$$

So the eigenvalues of $A$ is $\lambda_1 = 3$, $\lambda_2 = 2$. Now we can compute the corresponding eigenvectors are

$$v_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}, \quad v_2 = \begin{bmatrix} 1 \\ -2 \end{bmatrix}.$$

Thus

$$\vec{x}_1 = \begin{bmatrix} e^{3t} \\ -e^{3t} \end{bmatrix}$$

and

$$\vec{x}_2 = \begin{bmatrix} e^{2t} \\ -2e^{2t} \end{bmatrix}$$

are solutions of the system.

Let us verify now that $x_1$ and $x_2$ are linearly independent. Their Wronskian is

$$W(t) = \det \begin{bmatrix} e^{3t} & e^{2t} \\ -e^{3t} & -2e^{2t} \end{bmatrix} = e^{3t}(-2e^{2t}) - e^{2t}(-e^{3t}) = -e^{5t} \neq 0,$$

hence the solutions are linearly independent. Since the system is $2 \times 2$, it admits exactly two linearly independent solutions. We conclude that the general solution is

$$\vec{x} = c_1\vec{x}_1 + c_2\vec{x}_2.$$