**MA26500: EXAM I** **February 23, 2016**

Instructor: Yuanzhen Shao

**NAME:** ______________________ **PUID:**______________________

**Section Number:** ______________________ **Class Time:**______________________

(1) No calculators are allowed.

(2) No portable electronic devices.

(3) There are 10 problems. Each problem is worth 10 points.

(4) The score is accumulative and the maximum is 100.

**1.** Let $A_{ij}$ be the cofactor of the element $a_{ij}$ of the matrix $A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}$ with $\det(A) = 5$. Then the value of the expression $a_{11}A_{11} + a_{12}A_{12} + a_{21}A_{21} + a_{22}A_{22}$ is equal to

A. 0

B. 5

C. 10

D. 15

E. Undetermined by the information given above.

**2.** $A$ is a $3 \times 3$ matrix and $AX = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}$ has infinitely many solutions. Then $A$ satisfies

A. $A$ is nonsingular.

B. $A$ is symmetric.

C. The homogeneous system $AX = 0$ only has trivial solution.

D. The homogeneous system $AX = 0$ has non-trivial solutions.

E. None of the above.

**3.** Determine which one of the following expressions is the general solution to the inhomogeneous system of equations

$$\begin{cases} x_1 + x_2 - 2x_3 + 4x_4 = 5 \\ 2x_1 + 2x_2 - 3x_3 + x_4 = 3 \\ 3x_1 + 3x_2 - 4x_3 - 2x_4 = 1 \end{cases}$$

A. $\begin{bmatrix} -9 \\ 0 \\ -7 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} -9 \\ 0 \\ -7 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 10 \\ 0 \\ 7 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} -11 \\ 2 \\ -7 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}$

D. $s \begin{bmatrix} 1 \\ 1 \\ 0 \\ -2 \end{bmatrix} + t \begin{bmatrix} 0 \\ 0 \\ 1 \\ -4 \end{bmatrix}$

E. No solution.

**4.** The determinant of the matrix

$$A = \begin{bmatrix} 1 & 3 & -1 & 2 \\ 5 & 1 & 5 & 4 \\ 7 & 0 & 8 & 5 \\ 1 & 5 & 3 & 2 \end{bmatrix}$$

is

A. -1

B. 0

C. 1

D. 15

E. 2

**5.** If $A = \begin{bmatrix} 0 & 1 & 1 \\ -1 & 2 & -1 \\ 1 & 0 & 2 \end{bmatrix}$, then the **sum** of the entries in the third row of $A^{-1}$ is

A. -2

B. -1

C. 0

D. 1

E. 2

**6.** Find all the values of $k$ for which the system

$$\begin{cases} kx & + & y & + & z & = & 1 \\ 3x & + & (k+2)y & - & z & = & 5 \\ 2x & + & 2y & + & 2z & = & k+1 \end{cases}$$

has no solution. You may use the fact that $\det \begin{bmatrix} k & 1 & 1 \\ 3 & k+2 & -1 \\ 2 & 2 & 2 \end{bmatrix} = 2(k-1)(k+3)$.

A. $k = 1$

B. $k = 1, -3$

C. $k \neq 1, -3$

D. $k \neq 1$

E. $k = -3$

**7.** $A$ is an $m \times n$ matrix and $\mathbf{b}$ is an $m \times 1$ vector. $A\mathbf{x} = \mathbf{b}$ has no solution. Consider the following statements:

(i) $m < n$

(ii) $n < m$

(iii) the rank of $A < n$

(iv) the rank of $A < m$

Which **must** be true?

A. only (ii)

B. only (iv)

C. only (i) and (iii)

D. only (iii)

E. None of the statements has to be true.

**8.** For what value(s) of $\lambda$, can $\begin{bmatrix} 1 \\ 2 \\ \lambda \end{bmatrix}$ be expressed as a linear combination of $\left\{ \begin{bmatrix} 2 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 2 \\ 7 \end{bmatrix}, \begin{bmatrix} 1 \\ -1 \\ -4 \end{bmatrix}, \begin{bmatrix} 1 \\ 4 \\ 11 \end{bmatrix} \right\}$.

A. $\lambda = -2, 1$

B. $\lambda = 5, 6$

C. $\lambda = 1$

D. $\lambda = 0, 1$

E. $\lambda = 5$

**9.** Which of the following sets $S$ are subspaces of the given vector space $V$?

(i) $S = \{A \in V : \text{Tr}(A) = 0\}$, $V = M_3(\mathbb{R}) = \{3 \times 3 \text{ matrices with real entries}\}$

(ii) $S = \{f(t) = at^2+bt+c : a-2b+c = 1\}$, $V = \mathbb{P}_2 = \{\text{all polynomials with degree no more than } 2\}$

(iii) $S = \{A \in V : A^2 + A = 0\}$, $V = M_2(\mathbb{R}) = \{2 \times 2 \text{ matrices with real entries}\}$

(iv) $S = \{(x, y, z) \in V : 3x - y = 7z\}$, $V = \mathbb{R}^3$

(v) $S = \{$ solutions to the equation $\begin{bmatrix} 1 & 2 & -5 \\ 11 & -1 & 0 \\ 9 & -5 & 10 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} -1 \\ 12 \\ 14 \end{bmatrix} \}$, $V = \mathbb{R}^3$

A. only (iv)

B. only (i) and (iv)

C. only (i), (iii) and (v)

D. only (iii), (iv) and (v)

E. All of the above.

**10.** For an $n \times n$ matrix $A$, which of the following are true?

(i) $AA^T$ is a symmetric matrix, and $-A^TA$ is a skew-symmetric matrix.

(ii) If $A$ is both symmetric and skew-symmetric, then it is the zero matrix.

(iii) If $n$ is odd and $A$ is skew-symmetric, then $A$ is singular.

A. only (i)

B. only (i) and (ii)

C. only (ii) and (iii)

D. only (i), (ii) and (iii)

E. None of the above.