(1) C. $|A| = a_{11}A_{11} + a_{12}A_{12} = 5$

$= a_{21}A_{21} + a_{22}A_{22} = 5$

$$a_{11}A_{11} + a_{12}A_{12} + a_{21}A_{21} + a_{22}A_{22} = 2|A| = 10$$

(2) D

If A is nonsingular. $AX = \begin{bmatrix} 0 \\ \vdots \\ 0 \end{bmatrix}$ has a unique solution.

$\Updownarrow$

$AX = 0$ has only trivial solution.

(5) C

$$|A| = \begin{vmatrix} 0 & 1 & 1 \\ 0 & 2 & 1 \\ 1 & 0 & 2 \end{vmatrix} = \begin{vmatrix} 1 & 1 \\ 2 & 1 \end{vmatrix} = -1$$

$$A_{13} = \begin{vmatrix} -1 & 2 \\ 1 & 0 \end{vmatrix} = -2$$

$$A_{23} = -\begin{vmatrix} 0 & 1 \\ 1 & 0 \end{vmatrix} = 1$$

$$A_{33} = \begin{vmatrix} 0 & 1 \\ -1 & 2 \end{vmatrix} = 1$$

$$a^{-1}_{31} + a^{-1}_{32} + a^{-1}_{33} = \frac{-2+1+1}{-1} = 0$$

3. B

$$\left[\begin{array}{cccc|c} 1 & 1 & -2 & 4 & 5 \\ 2 & 2 & -3 & 1 & 3 \\ 3 & 3 & -4 & -2 & 1 \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 1 & 0 & -10 & -9 \\ 0 & 0 & 1 & -7 & -7 \\ 0 & 0 & 0 & 0 & 0 \end{array}\right]$$

free variables

$$X = \begin{bmatrix} -9 \\ 0 \\ -7 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 1 \\ 0 \\ 0 \end{bmatrix} + t \begin{bmatrix} 10 \\ 0 \\ 7 \\ 1 \end{bmatrix}$$

4. B

$$A \sim \begin{bmatrix} 1 & 3 & -1 & 2 \\ 0 & -7 & 5 & -3 \\ 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 0 \end{bmatrix} \quad \text{rank } A < 4 \Rightarrow \det A = 0$$

6. E

If $k \neq 1, -3$, this system has a unique solution

When $k = 1$

$$\left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 3 & 3 & -1 & 5 \\ 2 & 2 & 2 & 2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 0 & 0 & -4 & 2 \\ 0 & 0 & 0 & 0 \end{array}\right] \quad \text{infinitely many solutions}$$

When $k = -3$

$$\left[\begin{array}{ccc|c} -3 & 1 & 1 & 1 \\ 3 & -1 & -1 & 5 \\ 2 & 2 & 2 & -2 \end{array}\right] \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & -1 \\ 3 & -1 & -1 & 5 \\ 0 & 0 & 0 & 6 \end{array}\right] \quad \text{no solution}$$

7. B

(i) (ii) Counter example : $0 \cdot x = 1$

(iii) Counterexample : $\begin{cases} x = 1 \\ y = 1 \\ 0 = 1 \end{cases}$

⑧ E

$$\left[\begin{array}{cccc|c} 2 & -1 & 1 & 1 & 1 \\ 1 & 2 & -1 & 4 & 2 \\ 1 & 7 & -4 & 11 & \lambda \end{array}\right] \sim \left[\begin{array}{cccc|c} 1 & 2 & -1 & 4 & 2 \\ 0 & 5 & -3 & 7 & 3 \\ 0 & 0 & 0 & 0 & \lambda-5 \end{array}\right]$$

This system is consistent if and only if $\lambda = 5$.

⑨ B

(ii) $f_1(t) = t^2$, $f_2(t) = 1 \in S$. But $f_1 + f_2$ is not in S.

(iii) Consider $A = \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \in S$. But $2A$ is not in S.

⑩ C

(ii) $\left.\begin{array}{l} A \text{ sym} \Rightarrow A = A^T \\ A \text{ skew sym} \Rightarrow A^T = -A \end{array}\right\} \Rightarrow A = -A \Rightarrow A = 0$

(iii) $|A| = |A^T| \overset{\text{skew sym}}{=} |-A| = (-1)^n |A| \overset{n \text{ odd}}{=} -|A|$

$\Rightarrow |A| = 0 \Rightarrow A$ singular

(i) $(-AA^T)^T = -(A^T)^T A^T = -AA^T$ So $-AA^T$ is sym.