EXAMPLES OF SECTIONS 1.3

Question 1. Identity the matrix representation of the following linear system

$$\begin{cases} x + 3y + 2z = 2 \\ 2x + 7y + 7z = -1 \\ 2x + 5y + 2z = 7 \end{cases}$$

Question 2. Compute AB if.

$$A = \begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \\ 2 & -3 & 5 \end{bmatrix}$$

and

$$B = \begin{bmatrix} 7 & -4 & 3 \\ 1 & 5 & -2 \\ 0 & 3 & 9 \end{bmatrix}.$$

SOLUTIONS.

1. From the original system we read off

$$\begin{bmatrix} 1 & 3 & 2 \\ 2 & 7 & 7 \\ 2 & 5 & 2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2 \\ -1 \\ 7 \end{bmatrix}$$

The augmented coefficient matrix is

$$\begin{bmatrix} 1 & 3 & 2 & \vdots & 2 \\ 2 & 7 & 7 & \vdots & -1 \\ 2 & 5 & 2 & \vdots & 7 \end{bmatrix}$$

2. Let us compute AB by using a method different from the definition we learnt in class. First note that

$$AB = A \begin{bmatrix} c_1 & c_2 & c_3 \end{bmatrix} = \begin{bmatrix} Ac_1 & Ac_2 & Ac_3 \end{bmatrix},$$

where c_i denotes the i-th column of B. This means that AB can be obtained by computing the product of A with each column of B. Recall that the product of A with a column vector can be computed as follows:

$$\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 7 \\ 1 \\ 0 \end{bmatrix} = 7 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + 1 \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} + 0 \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} 7 \\ 23 \\ 11 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} -4 \\ 5 \\ 3 \end{bmatrix} = -4 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} + 5 \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} + 3 \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -13 \\ 10 \\ -8 \end{bmatrix},$$

$$\begin{bmatrix} 1 & 0 & -3 \\ 3 & 2 & 4 \\ 2 & -3 & 5 \end{bmatrix} \begin{bmatrix} 3 \\ -2 \\ 9 \end{bmatrix} = 3 \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix} - 2 \begin{bmatrix} 0 \\ 2 \\ -3 \end{bmatrix} + 9 \begin{bmatrix} -3 \\ 4 \\ 5 \end{bmatrix} = \begin{bmatrix} -24 \\ 41 \\ 57 \end{bmatrix}.$$

Therefore

$$AB = \begin{bmatrix} 7 & -13 & -24 \\ 23 & 10 & 41 \\ 11 & -8 & 57 \end{bmatrix}.$$