

## EXERCISES OF SECTIONS 5.3, 5.4, 5.5

**Question 1.** Which of the following statements are true?

- (i) Every Euclidean space has an orthonormal basis.
- (ii) Any orthogonal set is linearly independent.
- (iii) Any orthonormal set is linearly independent.
- (iv)  $A$  is an  $n \times n$  matrix and  $A^T A = I_n$ . Then the columns of  $A$  forms a basis for  $\mathbb{R}^n$ .

**Question 2.**  $W = \text{span}\{(2, 1, 3k, 4), (0, k - 1, 4, -8), (0, 0, 2, 1), (0, 0, k, 4)\}$ . Find all values of  $k$  such that  $\dim W^\perp = 0$ .

**Question 3.**  $W = \text{span}\{(1, 0, 1, 2), (0, 1, 1, 2)\}$ . Let

$$v = (1, 1, 1, -1), \quad u = (1, 0, 0, 0).$$

- (i) Find the closest points in  $W$  to  $v$  and  $u$ .
- (ii) Find the distance of  $v$  and  $u$  to  $W$ .