EXERCISES OF SECTIONS 5.3, 5.4, 5.5

Question 1. Which of the following statements are true?

(i) Every Euclidean space has an orthonormal basis.

(ii) Any orthogonal set is linearly independent.

(iii) Any orthonormal set is linearly independent.

(iv) A is an $n \times n$ matrix and $A^T A = I_n$. Then the columns of A forms a basis for $\mathbb{R}^n$.

Question 2. $W = \text{span}\{(2, 1, 3k, 4), (0, k - 1, 4, -8), (0, 0, 2, 1), (0, 0, k, 4)\}$. Find all values of k such that $\dim W^\perp = 0$.

Question 3. $W =$ the plane $x + y - z = 0$. Find a basis for $W^\perp$.

Question 4. $W = \text{span}\{(1, 0, 1, 2), (0, 1, 1, 2)\}$. Let

$$v = (1, 1, 1, -1), \quad u = (1, 0, 0, 0).$$

(i) Find the closest points in W to v and u.

(ii) Find the distance of v and u to W.