EXERCISES OF CHAPTER 5

Question 1. For what value of k is $(1, 2, -1)$ orthogonal to $(k^3, k^2, 1)$.

Question 2. Suppose the vector $v = (a, b, c, d)$ is orthogonal to $(1, 0, -1, 1)$, $(1, -1, 0, -1)$ and $(0, 1, 0, 2)$. Then which of one of a, b, c, d must always be zero:

A. a

B. b

C. c

D. d

E. None of them.

Question 3. Let W be the subspace of $\mathbb{R}^3$ spanned by $(1, 2, 1), (0, k, 1), (2, 4, k)$. For what value of k, is the dimension of $W^\perp$ nonzero?

Question 4. Consider the subspace W of $\mathbb{R}^4$ spanned by $(1, 1, 1, 1)$, $(0, 0, 3, 1)$, $(-1, -1, 2, 0)$, $(1, 1, 4, 2)$. What is the dimension of $W^\perp$?

Question 5. Let W denote the vector space spanned by the vectors

$$u_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \quad u_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \\ 2 \end{bmatrix}$$

and let $v = \begin{bmatrix} 2 \\ 1 \\ 3 \\ 6 \end{bmatrix}$. Find the distance from v to W.

Question 6. Let $S = \{e_1, e_2, e_3\}$ be the standard basis for $\mathbb{R}^3$. Assume that we are given an inner product $(\cdot, \cdot)$ on $\mathbb{R}^3$ such that the matrix of this inner product with respect to the standard basis S is

$$C = \begin{bmatrix} 3 & 3 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 4 \end{bmatrix}.$$

Under this inner product, what is the angle between e_1 and e_2?