THE PROOFS FOR PROPERTY (5) OF MATRIX MULTIPLICATION IN SECTION 1.3

Question 1. Prove $(AB)^T = B^T A^T$. Here A is an $m \times n$ matrix and B is of size $n \times p$.

Proof. Let $C = [c_{ij}] = (AB)^T$. Then

$$c_{ij} = (AB)_{ji} = \sum_{k=1}^{n} a_{jk} b_{ki} = \sum_{k=1}^{n} a_{kj}^T b_{ik}^T = \sum_{k=1}^{n} b_{ik}^T a_{kj}^T = (B^T A^T)_{ij},$$

where $(AB)_{ji}$ denotes the (i, j)-th entry of AB, $(B^T A^T)_{ij}$ is (i, j)-th entry of $(B^T A^T)$. a_{ij}^T and b_{ij}^T stand for the (i, j)-th entry of A^T and B^T respectively. We have used the fact

$$a_{ij} = a_{ji}^T$$

in the above equality. $\square$