

PROOF FOR THE UNIQUENESS OF MATRIX INVERSE

Example 1. *Suppose A, B, C are all $n \times n$ matrices, and*

$$BA = AB = I_n, \quad CA = AC = I_n.$$

Prove $B = C$.

Solution.

$$B = BI_n = B(AC) = (BA)C = I_n C = C.$$

In the third step, we used $I_n = AC$, and in the fourth step, we have used the associativity of matrix multiplication. ◀