EXAMPLES OF SECTIONS 2.6

Question 1. Find the inverse of

$$A = \begin{bmatrix} 3 & 5 & 6 \\ 2 & 4 & 3 \\ 2 & 3 & 5 \end{bmatrix}$$

Question 2. Solve the system

$$A\vec{x} = \vec{b},$$

where A is the matrix of question 1 and

$$\vec{b} = \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix},$$

by using the inverse of A.

SOLUTIONS.

1. Write

$$\left[\begin{array}{ccc:ccc} 3 & 5 & 6 & 1 & 0 & 0 \\ 2 & 4 & 3 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{array}\right]$$

and apply Gauss-Jordan elimination.

$$\left[\begin{array}{ccc:ccc} 3 & 5 & 6 & 1 & 0 & 0 \\ 2 & 4 & 3 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{A_{21}(-1)} \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & -1 & 0 \\ 2 & 4 & 3 & 0 & 1 & 0 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{A_{32}(-1)}$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 \\ 2 & 3 & 5 & 0 & 0 & 1 \end{array}\right] \underset{\sim}{A_{13}(-2)} \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 \\ 0 & 1 & -1 & -2 & 2 & 1 \end{array}\right]$$

$$\underset{\sim}{A_{23}(-1)} \left[\begin{array}{ccc:ccc} 1 & 1 & 3 & 1 & -1 & 0 \\ 0 & 1 & -2 & 0 & 1 & -1 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right] \begin{array}{c} A_{32}(2) \\ \underset{\sim}{A_{31}(-3)} \end{array}$$

$$\left[\begin{array}{ccc:ccc} 1 & 1 & 0 & 7 & -4 & -6 \\ 0 & 1 & 0 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right] \underset{\sim}{A_{21}(-1)} \left[\begin{array}{ccc:ccc} 1 & 0 & 0 & 11 & -7 & -9 \\ 0 & 1 & 0 & -4 & 3 & 3 \\ 0 & 0 & 1 & -2 & 1 & 2 \end{array}\right]$$

So

$$A^{-1} = \left[\begin{array}{ccc} 11 & -7 & -9 \\ -4 & 3 & 3 \\ -2 & 1 & 2 \end{array}\right].$$

2.

$$\vec{x} = A^{-1}\vec{b} = \left[\begin{array}{ccc} 11 & -7 & -9 \\ -4 & 3 & 3 \\ -2 & 1 & 2 \end{array}\right] \left[\begin{array}{c} 0 \\ 2 \\ 0 \end{array}\right] = \left[\begin{array}{c} -14 \\ 6 \\ 2 \end{array}\right].$$