INSTRUCTIONS:

1. **Do not open the exam booklet until you are instructed to do so.**
2. **Before you open the booklet fill in the information below and use a # 2 pencil to fill in the required information on the scantron.**
3. **MARK YOUR TEST NUMBER ON YOUR SCANTRON**
4. Once you are allowed to open the exam, make sure you have a complete test. There are 7 different test pages including this cover page.
5. Do any necessary work for each problem on the space provided or on the back of the pages of this test booklet. No extra paper is allowed. Circle your answers in this test booklet.
6. There are eleven problems, each problem is worth 9 points and everyone gets one point. The maximum possible score is 100 points. No partial credit.
7. Use a # 2 pencil to fill in the answers on your scantron
8. After you have finished the exam, hand in your scantron and your test booklet to one of the proctors.

RULES REGARDING ACADEMIC DISHONESTY:

1. Do not leave the exam room during the first 20 minutes of the exam.
2. If you do not finish your exam in the first 50 minutes, you must wait until the end of the exam period to leave the room.
3. Do not seek or obtain any kind of help from anyone to answer questions on this exam. If you have questions, consult only your instructor.
4. Do not look at the exam of another student. You may not compare answers with anyone else or consult another student until after you have finished your exam, handed it in to your instructor and left the room.
5. Do not consult notes, books, calculators.
6. Do not handle phones or cameras, or any electronic device until after you have finished your exam, handed it in to your instructor and left the room.
7. After time is called, the students have to put down all writing instruments and remain in their seats, while the proctors will collect the scantrons and the exams.
8. Anyone who violates these instructions will have committed an act of academic dishonesty. Penalties for academic dishonesty may include an F in the course. All cases of academic dishonesty will be reported immediately to the Office of the Dean of Students.

I have read and understand the above statements regarding academic dishonesty:

STUDENT NAME: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

STUDENT SIGNATURE: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

STUDENT ID NUMBER: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

SECTION NUMBER ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

RECITATION INSTRUCTOR: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

1. **1.** Let $y(x)$ be the solution of the initial value problem

$$x^2 y' + 2xy = 4x^3,$$
$$y(1) = 2.$$

Then $y(2)$ (the value of $y(x)$ when $x = 2$) is equal to

A. 4

B. $\dfrac{82}{9}$

C. $\dfrac{13}{4}$

D. $\dfrac{17}{4}$

E. 20

2. The population of a cell culture grows according to the rule

$$\frac{dP}{dt} = kP,$$

where $P = P(t)$ is the population at time t. If the initial number has doubled in 3 hours, how many times the initial number will be present after 7 hours?

A. $2^{\frac{7}{3}}$

B. $2^{\frac{5}{4}}$

C. $2^{\frac{7}{2}}$

D. 2^7

E. $2^{\frac{3}{2}}$

1. **1.** Let $y(x)$ be the solution of the initial value problem
$$x^2 y' + 2xy = 4x^3,$$
$$y(1) = 2.$$

The equation can be written as

$$y' + \frac{2}{x} y = 4x \longrightarrow \textcolor{red}{\textit{linear equation}}$$

Then $\mu = e^{\int \frac{2}{x} dx} = x^2$. Multiplying by μ, the equation becomes

$$(x^2 y)' = 4x^3 \longrightarrow \textcolor{blue}{\text{Rmk: This was the original}}$$
$$\textcolor{blue}{\text{form!}}$$

Integrate to get

$$x^2 y = x^4 + c$$

With initial condition $x = 1$, $y = 2$ we get

$$2 = 1^4 + c \implies c = 1$$

Thus
$$y = \frac{1}{x^2} (x^4 + 1)$$

$$\boxed{y(2) = \frac{1}{4} (16 + 1) = \frac{17}{4}}$$ $\textcolor{red}{\text{(D)}}$

2. The population of a cell culture grows according to the rule

$$\frac{dP}{dt} = kP,$$

where $P = P(t)$ is the population at time t. If the initial number has doubled in 3 hours, how many times the initial number will be present after 7 hours?

The general solution is

$$P = P_0 e^{kt}$$

Then $P(3) = 2P(0) \iff P_0 e^{3k} = 2P_0$

$$\iff 3k = \ln(2) \implies k = \frac{1}{3}\ln(2)$$

Next

$$\frac{P(7)}{P(0)} = e^{7k} = e^{\frac{7}{3}\ln(2)} = 2^{\frac{7}{3}}$$

$$\boxed{\frac{P(7)}{P(0)} = 2^{\frac{7}{3}}}$$

Ⓐ

3. The function $y(x)$ solves

$$y'' = y',$$
$$y(0) = 0 \quad y'(0) = 2.$$

Then $y(1)$ is equal to:

A. $y(1) = 2e - 2$

B. $y(1) = e - 1$

C. $y(1) = 3e - 3$

D. $y(1) = 3e - 2$

E. $y(1) = e - 2$

4. The function $y(x)$ solves the initial value problem

$$y'(x) + \frac{1}{x}y = 2x^4y^4,$$
$$y(1) = 1.$$

Then $y(2)$ is equal to

A. $y(2) = 2$

B. $y(2) = 4$

C. $y(2) = -\frac{1}{4}$

D. $y(2) = \frac{1}{8}$

E. $y(2) = -3$

3. The function $y(x)$ solves

$$y'' = y',$$
$$y(0) = 0 \quad y'(0) = 2.$$

Then $y(1)$ is equal to:

Set $v = y'$. Then v solves

$$v' = v \quad \Rightarrow \quad v = c_1 e^x$$

The initial condition is $v(0) = y'(0) = 2$. Thus

$$v = 2e^x$$

Next $y = \int v(x) \, dx = 2e^x + c_2$.
Since $y(0) = 0$ we get

$$c_2 = -2$$

$$\Rightarrow \quad y(x) = 2e^x - 2$$

$$\boxed{y(1) = 2e - 2}$$

Ⓐ

4. The function $y(x)$ solves the initial value problem

$$y'(x) + \frac{1}{x}y = 2x^4 y^4,$$
$$y(1) = 1.$$

Then $y(2)$ is equal to

This is a Bernoulli equation. Write it as

$$y^{-4}y' + \frac{1}{x}y^{-3} = 2x^4$$

Set $u = y^{-3} \implies u' = -3y^{-4}$. The equation becomes

$$-\frac{1}{3}u' + \frac{1}{x}u = 2x^4$$

$$\iff u' - \frac{3}{x}u = -6x^4 \longrightarrow \text{linear}$$

Then $\mu = e^{-\int \frac{3}{x}dx} = \frac{1}{x^3}$. We get

$$\left(\frac{1}{x^3}u\right)' = -6x$$

Integrate: $\frac{1}{x^3}u = -3x^2 + c_1$

$$u = -3x^5 + c_1 x^3$$

$$\implies \boxed{y = \left(-3x^5 + c_1 x^3\right)^{-1/3}}$$

Initial condition

$y(1) = 1 \iff (-3 + c_1)^{-\frac{1}{3}} = 1$

$\iff c_1 = 4$

Thus $y(x) = (-3x^5 + 4x^3)^{-\frac{1}{3}}$

we get

$y(2) = (-3 \times 32 + 32)^{-\frac{1}{3}}$

$= (-64)^{-\frac{1}{3}}$

$\boxed{y(2) = \frac{-1}{4}}$ ©