

MA 262 - DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

REVIEW PROBLEMS - MIDTERM 1

Problem 1. Find the general solution to the following equation:

$$y' = \frac{y}{x \ln(x)}.$$

Problem 2. Solve the equation

$$(3x^2 - 2xy + 2) dx + (6y^2 - x^2 + 3)dy = 0.$$

Problem 3. Find the general solution to

$$(x - 2y)dx + 4xdy = 0.$$

Problem 4. Solve the equation

$$y' - \frac{2}{x}y = x^2y^2$$

Problem 5. A tank originally contains 100 gal of fresh water. Then water containing 1 lb of salt per 2 gallon is poured into the tank at a rate of 2 gal/min, and the mixture is allowed to leave at the same rate. Find the quantity of salt in the tank after 50mn.

Problem 6. Let $\{\mathbf{v}_1, \dots, \mathbf{v}_4\}$ be the family of vectors defined by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} 5 \\ 6 \\ 7 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 9 \\ 10 \\ 11 \end{bmatrix}, \quad \mathbf{v}_4 = \begin{bmatrix} 13 \\ 14 \\ 15 \end{bmatrix}.$$

Exhibit a maximal number of linearly independent vectors in this family.

Problem 7. Let $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ be the family of vectors defined by

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \quad \mathbf{v}_2 = \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \quad \mathbf{v}_3 = \begin{bmatrix} 3 \\ k \\ -9 \end{bmatrix}.$$

Determine the values of k such that \mathbf{v}_3 is in $\text{Span}\{\mathbf{v}_1, \mathbf{v}_2\}$.

Problem 8. Determine the solution set for the following system:

$$\begin{array}{rclcl} 5x_1 & -6x_2 & +x_3 & = & 4 \\ 2x_1 & -3x_2 & +x_3 & = & 1 \\ 4x_1 & -3x_2 & -x_3 & = & 5 \end{array}$$

Problem 9. Discuss the number of solutions to the following system according to the values of a and b .

$$\begin{array}{rclcrcl} x_1 & +x_2 & -2x_3 & = & 4 \\ 3x_1 & +5x_2 & -4x_3 & = & 16 \\ 2x_1 & +3x_2 & -ax_3 & = & b \end{array}$$

Problem 10. Let $T : \mathbb{R}^2 \rightarrow \mathbb{R}^4$ be a linear transformation such that

$$T(-1, 1) = (1, 0, -2, 2), \quad T(1, 2) = (-3, 1, 1, 1).$$

Find $T(1, 0)$ and $T(0, 1)$.

Problem 11. Discuss

- Fall 18, Midterm 1, Problem 8.
- Fall 17, Midterm 1, Problem 7.
- Spring 17, Midterm 1, Problem 7.