MA 262 - DIFFERENTIAL EQUATIONS AND LINEAR ALGEBRA

REVIEW PROBLEMS - MIDTERM 2 - SPRING 19

Problem 1. Let A and B be defined by

$$A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & -2 & 0 \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}.$$

Compute $\det(A^{-1}B^2)$.

Problem 2. Consider the following vectors in $\mathbb{R}^3$:

$$\mathbf{v}_1 = \begin{bmatrix} 1 \\ 0 \\ z \end{bmatrix}, \qquad \mathbf{v}_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}, \qquad \mathbf{v}_3 = \begin{bmatrix} 0 \\ 2 \\ 1 \end{bmatrix}.$$

Find the values of z such that $\{\mathbf{v}_1, \mathbf{v}_2, \mathbf{v}_3\}$ forms a basis of $\mathbb{R}^3$.

Problem 3. Let D be the subspace of $\mathbb{R}^3$ defined by

$$D = \{\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \mathbf{x}_3);\ \mathbf{x}_1 - 3\mathbf{x}_2 + 6\mathbf{x}_3 = 0\}.$$

Find the dimension of D.

Problem 4. For the matrix A defined by

$$A = \begin{bmatrix} 0 & 2 & -1 \\ -2 & 0 & 1 \\ -1 & 0 & 1 \end{bmatrix},$$

compute the sum of the eigenvalues.

Problem 5. Let M be the following matrix:

$$M = \begin{bmatrix} 1 & -2 \\ 1 & 3 \end{bmatrix}.$$

Find the eigenvalue related to the eigenvector

$$\mathbf{v} = \begin{bmatrix} -1 + i \\ 1 \end{bmatrix}.$$

Problem 6. Solve the initial value problem

$$y'' - y' - 2y = 0, \qquad y(0) = 2,\ y'(0) = 0.$$

Problem 7. Find the general form of the trial solution for the following problem:

$$y'' - 2y' + 2y = e^x \cos(x) - 3e^{2x}.$$

Problem 8. Consider the equation

$$y'' - 3y' + 2y = t^{-1}e^{3t}$$

We wish to find y_p thanks to the variation of parameter method. Compute the expression for u_1' in this context.

Problem 9. For the following equation:

$$y'' - \frac{1}{t}y' + \frac{1}{t^2}y = 0,$$

a fundamental solution is $y_1(t) = t$. We wish to find the general solution under the form $t\,v$. Find the differential equation for v.

Problem 10. A mass-spring system is governed by the equation

$$y'' + by' + 2y = 0$$

According to the values of b, determine if the system is periodic, overdamped, underdamped or critically damped.