#### Outline

#### 1 Introduction

#### 1.1 Basic probability structures

- 1.2 Buffon's needle
- 1.3 Convergence of functions

#### 2 Modes of convergence

- 2.1 Reviewing the modes of convergence
- 2.2 Results for P and  $L^p$  convergences
- 2.3 Results for almost sure convergence
- 2.4 Cases of inverse relations for modes of convergence
- 2.5 Inverse method for simulation
- 2.6 Results for convergence in distribution

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#### Probability space

#### Probability space: $(\Omega, \mathcal{F}, \mathbf{P})$ with

- $\Omega$  set
- $\mathcal{F} = \sigma$ -algebra or  $\tau$ -field
- P probability measure

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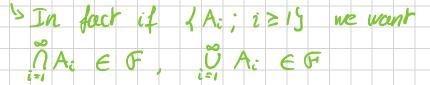
Prob space: (2, F, P)

## F: σ-algebra, i·e F is a collection of xubsets of JZ s.r.

# $(i) \mathcal{L} \in \mathcal{F}, \quad \phi \in \mathcal{D} \quad \text{or} \overline{A}$

# $(\alpha)$ If $A \in F$ , then $A^{c} \in F$

# (ill) If A, B E F, then A NB E F



# About P: P is a probability measure, $\mathcal{P}: \mathcal{F} \longrightarrow \mathcal{I}O_{i}\mathcal{I} \qquad s.r.$ (i) $P(\phi) = O$ $P(\chi) = I$ (ii) If $\{A_i; i \ge 1\}$ are disjoint, $\mathbb{P}(\bigcup_{i=1}^{\infty}A_{i}) = \sum_{i=1}^{\infty}\mathbb{P}(A_{i})$ (iii) $\mathcal{P}(A^{\circ}) = 1 - \mathcal{P}(A) + A \in \mathcal{F}$ Rml If AEF, we say that A is measurable

### Example 1 Tossing 2 dice. Then

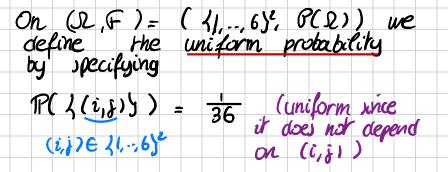
# $\Omega = \{1, ..., 6\} \times \{1, ..., 6\} = \{1, ..., 6\}^2$

# Since I is finite (or countable), we can take

# F=P(I)=Lall rubies of ly

# $= \langle \emptyset, \langle (1,1) \rangle, \langle 0,2 \rangle \dots \langle (1,1), (1,2) \rangle \dots \}$

#### Question: $|\mathcal{X}| = #$ elements in $\mathcal{R} = 36$ $|\mathcal{F}| = 2^{36}$



+ properties (i)-(ii))



F= Borel J-algebra, i.e. we declare that any internal (a,6) c [0,1] belongs to F

+ properties (i)-(ii)-(iii)

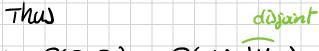
Uniform prob measure: If A = (a,b) Then

P(A)= b-a

Then use (i)-(ic)-(iii)

#### <u>Rma</u> It is important to consider only caustable unions. Otherwise one could write

## $\mathbb{P}(\{t\}) = \mathbb{P}([t,t]) = O$



# $I = P([0,1]) = P(\bigcup \{t\})$ $= Z P(\{t\}) = O$ $= C P(\{t\}) = O$

#### Complete probability space

Hypothesis: We assume that **P** is complete, i.e

$$A \in \mathcal{F}$$
 such that  $\mathbf{P}(A) = 0$ , and  $B \subset A$   
 $\Longrightarrow$   
 $B \in \mathcal{F}$  and  $\mathbf{P}(B) = 0$ .

Remark: A probability can always be completed

Image: Image:

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Simple examples (1)

#### Tossing 2 dice:

Ω = {1, 2, 3, 4, 5, 6}<sup>2</sup>
F = P(Ω)
P(A) = |A|/36

#### Uniform distribution on [0, 1]:

- $\Omega = [0, 1]$
- $\mathcal{F} = \mathcal{B}([0,1])$
- $\mathbf{P} = \lambda$ , Lebesgue measure

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