421 HOMEWORK

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ABSTRACT. Textbook: V. Chvatal, "Linear Programming".
grade: 30% midterm, 30% homework, 40% final.
homework collected each Thursday in class or in my office by 3pm.

Homework 1 (for week 2)**.**

(1) Find an example of an optimization problem on a region in $\mathbb{R}^2$ with a linear objective function f that is bounded on C but where there is no max in C. (This will by our theorem require a non-closed C). Prove that there is no max on C.
(2) Find an example of an optimization problem with a closed C where the max of f is not on the boundary and not at infinity. (By the theorem, this will require a non-linear f.) Prove that the max is where you claim it to be.
(3) Solve the following linear program graphically and explain why the max is taken where you claim it is.

$$\begin{aligned} x_1 &\leq 4 \\ 2x_2 &\leq 12 \\ 3x_1 + 2x_2 &\leq 18 \\ x_1, x_2 &\geq 0 \\ 3x_1 + 5x_2 &\to \max \end{aligned}$$

(4) Solve the following linear program graphically and explain why the max is taken where you claim it is.

$$\begin{aligned} x_2 &\leq 10 \\ x_1 + 2x_2 &\leq 15 \\ x_1 + x_2 &\leq 12 \\ 5x_1 + 3x_2 &\leq 45 \\ x_1, x_2 &\geq 0 \\ 10x_1 + 20x_2 &\to \max \end{aligned}$$

(5) The Southern Confederation of Kibbutzim (SCK) is a group of three kibbutzim (communal farming communities) in Israel. Overall planning is done by the Coordinating Technical Office.

The agricultural output for each kibbutz is limited by both the amount of available irrigable land and the quantity of water allocated

for irrigation by the Water Commissioner, a national governmental official. These data are given in the following table.

Kibbutz	Usable land (acres)	water allocation (acre feet)
1	400	600
2	600	800
3	300	375

The crops suited for this region include sugar beets, cotton, and sorghum, and these are the only ones considered for the upcoming season. These crops differ in their expected net return per acre, and their water needs. Additionally, the Ministry of Agriculture has set a maximum quota for the total acreage that can be used to each of these crops by the SCK as shown in the next table.

Crop	Max Quota (Acres)	Water consumption (Acre feet/Acre)	Net return ($/Acre)
sugar beets	600	3	1000
cotton	500	2	750
sorghum	325	1	250

Using the variables $x_1, \ldots, x_9$ to indicate how much of sugar beets, cotton and sorghum is grown by kibbutz 1,2 and 3 (see the table below), formulate a linear program whose optimal solution maximizes the total net return, but do not solve it.

kibbutz	1	2	3
beets	x_1	x_2	x_3
cotton	x_4	x_5	x_6
sorghum	x_7	x_8	x_9

Homework 2 (for week 3)**.**

(1) Farmer Jones has 100 acres of land to devote to wheat and corn and wishes to plan his planting to maximize the expected revenue. Jones has only \$800 in capital to apply to planting the crops, and it costs \$5 to plant an acre of wheat, and \$10 for an acre of corn. Their other activities leave the Jones family only 150 days of labor for the crops. Two days are required for each acre of wheat and one day for an acre of corn. Past experience indicates a return of \$80 for an acre of wheat and \$60 for an acre of corn. How should farmer Jones use his land?

(2) You are given the following information on steaks and potatoes:

nutrient	steak (grams)	potatoes (grams)	daily needs (grams)
carbohydrates	5	15	≥ 50
protein	20	5	≥ 40
fat	15	2	≤ 60
cost per serving	\$ 4	\$ 2	

Find the cheapest steak-and-potato diet fitting the nutritional requirements.

(Note: if you introduce slack, be careful you put it on the correct side of the inequality - it must be added to the smaller side!

Also, in this case, the origin "steak=0, potato=0" is not feasible. Because of this, you are given the feasible, non-yummy initial solution potato=30, steak=0. You therefore need to find first the correct table. What variables should be on the left? Those that are non-zero. This will be the potatoes, and 2 slack variables.)

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