Exercises 10.3

1. Let $\mathbf{V} = R^3$ with the standard inner product and let

$$\mathbf{S} = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3\} = \left\{ \begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Use routine **gschmidt** in MATLAB to obtain an orthonormal basis $\mathbf{T}$ and then find the coordinates of $\boldsymbol{x} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ relative to $\mathbf{T}$. Record the orthonormal basis and the coordinates of $\boldsymbol{x}$ below.

2. Let $\mathbf{V} = R^4$ with the standard inner product and let

$$\mathbf{S} = \{\boldsymbol{u}_1, \boldsymbol{u}_2, \boldsymbol{u}_3, \boldsymbol{u}_4\} = \left\{ \begin{bmatrix} -1 \\ 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix} \right\}.$$

Use routine **gschmidt** in MATLAB to obtain an orthonormal basis $\mathbf{T}$ and then find the coordinates of $\boldsymbol{x} = \begin{bmatrix} 4 \\ 0 \\ 2 \\ 1 \end{bmatrix}$ relative to $\mathbf{T}$. Record the orthonormal basis and the coordinates of $\boldsymbol{x}$ below.

3. Let $\mathbf{V} = R^4$ with the standard inner product and let

$$\mathbf{S} = \{\boldsymbol{u_1}, \boldsymbol{u_2}, \boldsymbol{u_3}, \boldsymbol{u_4}\} = \left\{ \begin{bmatrix} .5 \\ .5 \\ .5 \\ .5 \end{bmatrix}, \begin{bmatrix} .5 \\ .5 \\ -.5 \\ -.5 \end{bmatrix}, \begin{bmatrix} .5 \\ -.5 \\ -.5 \\ .5 \end{bmatrix}, \begin{bmatrix} .5 \\ -.5 \\ .5 \\ -.5 \end{bmatrix} \right\}.$$

a) Is **S** an orthonormal basis? Circle one: Yes No

Explain your answer.

b) In MATLAB form the matrix $\boldsymbol{T}$ whose columns are the vectors in **S**. Generate a random vector in R^4 using command **x = rand(4,1)** and then compute $\| \boldsymbol{x} \|$ and $\| \boldsymbol{Tx} \|$. How are the values of the norms related? Repeat the experiment for another arbitrary vector.

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4. Let $\boldsymbol{v_1} = \begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$ and $\boldsymbol{v_2} = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$. In MATLAB form the matrix **A = [v1 v2]** and then use command **gschmidt(A)**. Explain the meaning of the display generated.

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5. Let $\boldsymbol{A} = \begin{bmatrix} 1 & i & 0 \\ i & 0 & 1 \end{bmatrix}$.

a) In MATLAB use command **A′**. Record the result. **A′** = ____________

b) In MATLAB use command **C = A′∗A**. Record the result. **C** = ____________

c) What is the relation between $\boldsymbol{C}$ and $\boldsymbol{C'}$?

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d) Experiment with other complex matrices $\boldsymbol{A}$ to confirm or reject your answer in part c).

Circle one: confirmed not confirmed.

6. A complex matrix $\boldsymbol{A}$ is called Hermitian if it is equal to its conjugate transpose. The command $\mathbf{A}'$ gives the conjugate transpose in MATLAB .

a) How can you use MATLAB to determine if the matrix $\boldsymbol{A}$ below is Hermitian?

$$\boldsymbol{A} = \begin{bmatrix} 2 & 3-3i \\ 3+3i & 5 \end{bmatrix}$$

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b) Compute $\boldsymbol{r} = \boldsymbol{x}' * \boldsymbol{A} * \boldsymbol{x}$ for the complex vector below.

$$\boldsymbol{x} = \begin{bmatrix} i \\ 1-i \end{bmatrix} \qquad \boldsymbol{r} = \_\_\_\_\_\_\_\_\_\_\_\_\_\_$$

Is $\boldsymbol{r}$ a real number? (Circle one:) YES NO

c) Experiment with other complex vectors $\boldsymbol{x}$ to determine whether $\boldsymbol{x}'\boldsymbol{A}\boldsymbol{x}$ will always be a real number. (Circle one:)

Always a real number for this matrix $\boldsymbol{A}$. Not always a real number.

d) Experiment with another Hermitian matrix $\boldsymbol{A}$ and arbitrary vector $\boldsymbol{x}$ to see if $\boldsymbol{r} = \boldsymbol{x}' * \boldsymbol{A} * \boldsymbol{x}$ is always a real number.

(Circle one:) Always a real number. Not always a real number.

7. Let $\mathbf{V} = R^4$ with the standard inner product and let

$$\boldsymbol{v_1} = \begin{bmatrix} 3 \\ 1 \\ 2 \\ 0 \end{bmatrix}, \quad \boldsymbol{v_2} = \begin{bmatrix} 1 \\ -1 \\ -1 \\ 1 \end{bmatrix}, \quad \boldsymbol{v_3} = \begin{bmatrix} 0 \\ -2 \\ 1 \\ -1 \end{bmatrix}.$$

a) Find an orthonormal basis for R^4 containing scalar multiples of the vectors $\boldsymbol{v_1}$ and $\boldsymbol{v_2}$. Record your result below.

b) Find an orthonormal basis for R^4 containing scalar multiples of the vectors $\boldsymbol{v_1}$, $\boldsymbol{v_2}$, $\boldsymbol{v_3}$. Record your result below.

<< NOTES; COMMENTS; IDEAS >>