MA 265 – Midterm I **PRACTICE**

NAME: ____________________ Student ID: ____________________

Books, notes or any kind of calculator are NOT allowed.

Problem	Score
1 & 2	
3	
4	
5	
6	
Bonus	
Total	

Honor Pledge: I have neither received nor given aid on this exam.

Signature: ____________________

1. (5 pts) Let $A = \begin{bmatrix} a & b & c & d \\ x & e^x & \cos x & \sin x \\ 1 & 2 & 3 & 4 \end{bmatrix}$ and $B = \begin{bmatrix} \log x & 1 & 0 & 4 \\ 2 & 0 & 0 & 4 \\ -3 & -1 & 1 & 2 \end{bmatrix}$. Let $C = B^T A$ and c_{ij} denote the entries of C. What is c_{32}?

A. b

B. 2

C. c-3

D. 3-c

E. $B^T A$ is not well defined.

2. (5 pts) Given the linear system $A\mathbf{x} = \mathbf{b}$ with

$$A = \begin{bmatrix} 1 & 4 & 2 \\ 1 & 2 & 2 \\ -3 & 0 & -1 \end{bmatrix}, \quad \mathbf{x} = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

Suppose we already know $|A| = -10$. By **Cramer's rule**, $x_3 =$?

A. $-\frac{3}{5}$

B. $\frac{3}{5}$

C. -6

D. 6

E. 0

3. (20 pts) Given four vectors $\mathbf{v}_1 = \begin{pmatrix} -1 \\ 0 \\ 0 \\ 2 \end{pmatrix}$, $\mathbf{v}_2 = \begin{pmatrix} -1 \\ -1 \\ 0 \\ 1 \end{pmatrix}$, $\mathbf{v}_3 = \begin{pmatrix} 0 \\ 1 \\ -1 \\ 0 \end{pmatrix}$ and $\mathbf{u} = \begin{pmatrix} 0 \\ 0 \\ -1 \\ 3 \end{pmatrix}$,

is $\mathbf{u}$ a linear combination of $\mathbf{v}_1$, $\mathbf{v}_2$ and $\mathbf{v}_3$? If yes, find all possible coefficients.

4. (20 pts) Calculate

$$\begin{vmatrix} 1 & 2 & 3 & 4 \\ 1 & 0 & 3 & 0 \\ 0 & 1 & 2 & 0 \\ 1 & 0 & 0 & -3 \end{vmatrix}.$$

5. (20 pts) Consider $A = [a_{ij}] = \begin{bmatrix} 3 & 1 & 2 & 0 \\ x+1 & -1 & 0 & 0 \\ \cos x & 1 & 1 & 1 \\ 0 & x-1 & 0 & 0 \end{bmatrix}$.

(a) Find all possible values of x such that A is nonsingular.

(b) Find all possible values of x such that the cofactor of a_{14} is equal to 1.

6. (30 pts) Answer only TRUE or FALSE in the table for the following statements:

a. For nonsingular matrix A, if $\mathrm{adj}(A)$ is invertible, then the inverse of $\mathrm{adj}(A)$ must be $A/|A|$.

b. If $A^2 = \mathbf{0}$, then A is singular.

c. A must be a square matrix so that AA^T is well-defined and symmetric.

d. If a matrix is symmetric and also skew symmetric, then it must be a zero matrix.

e. $|-A| = |A|$.

f. $\begin{bmatrix} 0 & 1 & 0 & 2 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix}$ is in reduced row echelon form.

g. If A and B have the same null spaces, then A and B are row equivalent.

h. $\begin{vmatrix} a & b & c \\ x & y & 0 \\ z & 0 & 0 \end{vmatrix} = cyz.$

i. Any plane in $\mathbb{R}^3$ is a subspace of $\mathbb{R}^3$.

j. For a square matrix A, if the nonhomogeneous system $\mathbf{Ax} = \mathbf{b}$ has no solutions, then A is a singular matrix.

a	b	c	d	e	f	g	h	i	j

Solutions:

1. B.

2. A.

3. If $\mathbf{u}$ is a linear combination, then there are three numbers x_1, x_2 and x_3 such that $x_1\mathbf{v}_1 + x_2\mathbf{v}_2 + x_3\mathbf{v}_3 = \mathbf{u}$. Plug in the vectors then we get a system of four equations and three unknowns. Solve it by Gaussian elimination then we find there are no solutions. So the answer is negative.

4. 24.

5. (a) Using cofactor expansion along the last column, we get
$$|A| = 1 * (-1)^{3+4} \begin{vmatrix} 3 & 1 & 2 \\ x+1 & -1 & 0 \\ 0 & x-1 & 0 \end{vmatrix} = -2(x+1)(x-1).$$ Thus $|A| \neq 0$ implies $x \neq \pm 1$.

   (b) The cofactor of a_{14} is $(-1)^{4+1} \begin{vmatrix} x+1 & -1 & 0 \\ \cos x & 1 & 1 \\ 0 & x-1 & 0 \end{vmatrix} = x^2 - 1$. Thus $x^2 - 1 = 1$ gives us $x = \pm\sqrt{2}$.

6. a. T. Because $\operatorname{adj}(A)A = |A|I$.

   b. T. $A^2 = \mathbf{0} \Rightarrow |A^2| = 0 \Rightarrow |A|^2 = 0 \Rightarrow |A| = 0$.

   c. F. A could be any matrix.

   d. T.

   e. F. $|cA| = c^n|A|$.

   f. T.

   g. F. Counter example: $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ and $B = \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 1 & 0 \end{pmatrix}$, they have the same null space $\{\mathbf{0}\}$ but they have different sizes thus cannot be row equivalent.

   h. F. By cofactor expansion, it should $-cyz$.

   i. F. If the plane does not pass the origin, then the subset does not contain zero vector.

   j. T. If A is nonsingular, then A^{-1} exists thus $A^{-1}\mathbf{b}$ is a solution.