Practice Problems for Midterm II

The actual exam will be shorter and probably easier, and all problems will be multiple-choice.

1. Find all real numbers a such that the polynomial $3at^2 - a^2t + 2$ is in the span of $2t^2 + t + 3$, $t + 1$, and $3t^2 + t + 4$.

2. Find all real numbers a such that the vectors $[1, 2]$, $[a, 2]$, and $[2, a]$ span R_2.

3. Find all real numbers a such that the matrices $\begin{bmatrix} a & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 1 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ a & 0 \end{bmatrix}$ are linearly independent.

4. Find a basis for the subspace of R_5 spanned by $[5, 1, 4, 3, 2]$, $[0, 0, 1, 0, 0]$, $[0, 0, 0, 0, 2]$, $[10, 2, 5, 6, 4]$, and $[10, 2, 8, 6, 5]$.

5. Let $A = \begin{bmatrix} 3 & 3 & 2 & 5 \\ 0 & 3 & 1 & -2 \\ 1 & 2 & 1 & 1 \end{bmatrix}$.

   a. Find a basis for the column space of A.

   b. Find a basis for the null space of A.

   c. What is the rank of A?

   d. What is the nullity of A?

6. Suppose the matrix $A = \begin{bmatrix} a & b & c & d \\ e & f & g & h \\ i & j & k & l \\ m & n & p & q \end{bmatrix}$ has reduced row echelon form $\begin{bmatrix} 1 & 1 & 2 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$.

   a. Find the determinant of A.

   b. Find the rank of A.

   c. Find the nullity of A.

   d. Find a basis for the column space of A.

   e. Find a basis for the row space of A.

   f. Find a basis for the null space of A.

7. Let A be a 4×6 matrix whose nullspace is spanned by $\begin{bmatrix} 3 \\ 1 \\ 0 \\ 1 \\ 2 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ -2 \\ 3 \\ -1 \\ -1 \\ 2 \end{bmatrix}$.

   a. Find the rank of A.

   b. Is $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$ a solution to $Ax = 0$?

c. How many solutions are there for $Ax = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$?

8. Find a basis for the set of 3×2 matrices of the form $\begin{bmatrix} a+2c & b+c \\ a+b+3c & a+b+3c \\ 2a+b+5c & 3a+6c \end{bmatrix}$ where a, b and c are any real numbers.

9. Find all real numbers a such that the vectors $\begin{bmatrix} -8 \\ a \\ 0 \end{bmatrix}$ and $\begin{bmatrix} 2 \\ a \\ 5 \end{bmatrix}$ are orthogonal.

10. Let $\mathbf{u}$ and $\mathbf{v}$ be two vectors satisfying $\langle \mathbf{u}, \mathbf{v} \rangle = 0$, $\|\mathbf{u}\| = 2$, and $\|\mathbf{v}\| = 3$. What is $\|2\mathbf{u} - 3\mathbf{v}\|$?

11. Find the orthonormal set of vectors obtained by applying the GramSchmidt process to the three vectors $\begin{bmatrix} 1 \\ -1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 5 \\ 1 \\ 0 \end{bmatrix}$.

12. Let W be the subspace of R_3 spanned by $[1, 2, 3], [2, k, 3]$ and $[4, 5, k]$. Find all values of k such that the dimension of $W^\perp$ is 0.

13. Let W be the subspace of R_4 spanned by $\begin{bmatrix} 1 & 2 & 3 & 4 \end{bmatrix}, \begin{bmatrix} 5 & 6 & 7 & 8 \end{bmatrix}, \begin{bmatrix} 9 & 10 & 11 & 12 \end{bmatrix}, \begin{bmatrix} 0 & 0 & 0 & 1 \end{bmatrix}$. Find a basis for the orthogonal complement of W.

14. Let W be the subspace of R_4 spanned by the orthonormal set $\{[1/\sqrt{2}, 0, 0, 1/\sqrt{2}], [0, 0, 1, 0], [-1/\sqrt{2}, 0, 0, 1/\sqrt{2}]\}$. Suppose $[2, 1, 0, 4] = \mathbf{w} + \mathbf{v}$ with $\mathbf{w}$ in W and $\mathbf{v}$ in $W^\perp$. Find $\mathbf{w}$ and $\mathbf{v}$.

15. Let W be the subspace of R_3 with basis $\{[1, 1, 0], [0, 1, 1]\}$. Find the vector in W closest to $[2, 4, 0]$

16. Find the least squares solution to

$$\begin{bmatrix} 2 & 1 \\ 1 & 2 \\ 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}.$$

17. Find the least squares line $y = ax + b$ for the data table below

x	-3	0	1	2
y	1	0	1	2