

# Fall 2026 Graduate Mathematics Course Descriptions

Purdue University — Department of Mathematics

## MA 51400 Numerical Analysis

**Instructor:** Professor Steven Dong

**Course Number:** MA 51400 (CS 51400)

**Credits:** Three

**Meeting Time:** 8:30–9:20 AM Monday / Wednesday / Friday

### Catalog Description

Iterative methods for solving nonlinear equations; linear difference equations, applications to solution of polynomial equations; differentiation and integration formulas; numerical solution of ordinary differential equations; roundoff error bounds.

## MA 51900 Introduction To Probability

**Instructor:** Professor Yuan Gao

**Course Number:** MA 51900 (STAT 51900)

**Credits:** Three

**Meeting Time:** 4:30–5:45 PM Tuesday / Thursday

### Catalog Description

Algebra of sets, sample spaces, combinatorial problems, independence, random variables, distribution functions, moment generating functions, special continuous and discrete distributions, distribution of a function of a random variable, limit theorems.

## **MA 52300 Introduction to Partial Differential Equations**

**Instructor:** Professor Plamen Stefanov

**Course Number:** MA 52300

**Credits:** Three

**Meeting Time:** 12:00–1:15 PM Tuesday / Thursday

### **Catalog Description**

First order quasi-linear equations and their application to physical and social sciences; the Cauchy–Kovalevsky theorem; characteristics, classification, and canonical form of linear equations; equations of mathematical physics; study of the Laplace, wave and heat equations; methods of solution.

## **MA 53000 Functions Of A Complex Variable I**

**Instructor:** Professor Gregory Buzzard

**Course Number:** MA 53000

**Credits:** Three

**Meeting Time:** 9:30–10:20 AM Monday / Wednesday / Friday

### **Description**

This course will be a mathematically rigorous introduction to complex analysis. Topics will include complex-valued functions of one complex variable; differentiation and contour integration; Cauchy’s theorem; Taylor and Laurent series; residues; conformal mapping; special topics. This course is more mathematically rigorous than MA 52500 and will focus on ideas and proofs rather than applications.

### **Prerequisites**

A solid foundation in real analysis at the level of MA 50400.

### **Textbook**

*Complex Analysis* by Stein and Shakarchi.

## MA 53900 Probability Theory II

**Instructor:** Professor Jing Wang

**Course Number:** MA 53900 (STAT 53900)

**Credits:** Three

**Meeting Time:** 12:00–1:15 PM Tuesday / Thursday

### Catalog Description

Convergence of probability laws; characteristic functions; convergence to the normal law; infinitely divisible and stable laws; Brownian motion and the invariance principle.

## MA 54400 Real Analysis And Measure Theory

**Instructor:** Professor Nicholas James McCleerey

**Course Number:** MA 54400

**Credits:** Three

**Meeting Time:** 2:30–3:20 PM Monday / Wednesday / Friday

### Description

Sigma algebras, measures, Lebesgue integration, convergence theorems, outer measures and Caratheodory's construction, Hausdorff measure, modes of convergence, Egorov's theorem, Lusin's theorem, product measures, Fubini–Tonelli theorems, Lebesgue differentiation theorem, Hardy–Littlewood maximal function, bounded variation, absolute continuity,  $L^p$  spaces and some elements of functional analysis (Hilbert, Banach, and normed spaces), and time permitting, Fourier series and Parseval's theorem.

### Textbook

Mainly drawn from Folland's *Real Analysis: Modern Techniques and Their Applications* and Rudin's *Real and Complex Analysis*.

## MA 54500 Functions Of Several Variables And Related Topics

**Instructor:** Professor Monica Torres

**Course Number:** MA 54500

**Credits:** Three

**Meeting Time:** 11:30–12:20 PM Monday / Wednesday / Friday

### Description

Riesz Representation Theorem, weak convergence and compactness for Radon measures, weak convergence in  $L^p$ , weak compactness in  $L^p$ , differentiation of distributions, weak derivatives, Rademacher theorem for Lipschitz functions, Area and Coarea formulas, Sobolev spaces, approximation of Sobolev functions, compactness, traces of Sobolev functions, Sobolev Embedding Theorem, Laplace's equation, existence of weakly harmonic functions, regularity of weakly harmonic functions, functions of bounded variation in several variables, approximation and compactness, traces of functions of bounded variation and the Gauss–Green formula, sets of finite perimeter and the existence of minimal surfaces.

### References

1. William P. Ziemer (with contributions by Monica Torres), *Modern Real Analysis*, 2nd ed., Graduate Texts in Mathematics, Vol. 278, Springer, 2017.
2. Enrico Giusti, *Minimal Surfaces and Functions of Bounded Variation*, Monographs in Mathematics 80, Birkhäuser, 1984.
3. L. C. Evans and R. Gariepy, *Measure Theory and Fine Properties of Functions*, Studies in Advanced Mathematics, CRC Press, 1992.
4. William P. Ziemer, *Weakly Differentiable Functions*, Graduate Texts in Mathematics 120, Springer-Verlag.
5. Luigi Ambrosio, Nicola Fusco, and Diego Pallara, *Functions of Bounded Variation and Free Discontinuity Problems*, Oxford Mathematical Monographs, Oxford University Press, 2000.

## MA 55300 Introduction To Abstract Algebra

**Instructor:** Professor Saugata Basu

**Course Number:** MA 55300

**Credits:** Three

**Meeting Time:** 11:30–12:20 PM Monday / Wednesday / Friday

### **Catalog Description**

Group theory: Sylow theorems, Jordan–Hölder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

## **MA 55400 Linear Algebra**

**Instructor:** Professor Bernd Ulrich

**Course Number:** MA 55400

**Credits:** Three

**Meeting Time:** 3:30–4:20 PM Monday / Wednesday / Friday

### **Catalog Description**

Review of basics: vector spaces, dimension, linear maps, matrices, determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

## **MA 55700 Commutative Algebra I**

**Instructor:** Professor Takumi Murayama

**Course Number:** MA 55700

**Credits:** Three

**Meeting Time:** 1:30–2:45 PM Tuesday / Thursday

## Description

This course is an introduction to commutative algebra. Commutative algebra is the study of commutative rings and modules and has interactions with various fields of mathematics, including algebraic geometry, number theory, and several complex variables. Planned topics include the following:

- A review of commutative rings and modules. Spec and the Zariski topology.
- Localization. Integral extensions and integral closure. Noether normalization.
- Hilbert's Nullstellensatz and connections to affine algebraic geometry.
- Chain conditions, Noetherian and Artinian rings and modules. Tensor products and flatness.
- Primary decomposition. The Krull height theorem and dimension theory.
- Normal rings, discrete valuation rings, and Dedekind domains.
- The Artin–Rees lemma. Completions.

## Prerequisites

MA 55300 and MA 55400. MA 57100 is recommended.

## Textbook

Course notes will be provided, largely drawn from Melvin Hochster's Fall 2017 lecture notes on commutative algebra.

## Optional Texts

All texts listed below have free access options for Purdue students.

- *A term of commutative algebra* by Allen B. Altman and Steven L. Kleiman (available at doi:10.13140/RG.2.2.31866.62400).
- *Introduction to commutative algebra* by Michael F. Atiyah and Ian G. Macdonald (available at doi:10.1201/9780429493638 via the Purdue library).
- *Undergraduate commutative algebra* by Miles Reid (available for short term loan at <https://n2t.net/ark:/13960/s27c54tjnc0>).

## MA 56200 Introduction To Differential Geometry And Topology

**Instructor:** Professor David Ben McReynolds

**Course Number:** MA 56200

**Credits:** Three

**Meeting Time:** 9:30–10:20 AM Monday / Wednesday / Friday

### Catalog Description

Smooth manifolds; tangent vectors; inverse and implicit function theorems; submanifolds; vector fields; integral curves; differential forms; the exterior derivative; DeRham cohomology groups; surfaces in  $E^3$ ; Gaussian curvature; two dimensional Riemannian geometry; Gauss-Bonnet and Poincaré theorems on vector fields.

## MA 56500 Algebraic Geometry I

**Instructor:** Professor Jaroslaw Wlodarczyk

**Course Number:** MA 56500

**Credits:** Three

**Meeting Time:** 4:30–5:20 PM Tuesday / Thursday

### Description

This course is intended as the first part of a two-semester introduction to algebraic geometry. The subject studies geometric objects defined by systems of polynomial equations and develops algebraic tools for understanding their structure and behavior.

The course will focus primarily on the theory of algebraic varieties over algebraically closed fields, following Chapters I and II of Hartshorne's *Algebraic Geometry*. Toward the end of the semester, we will begin introducing the language of schemes and sheaves, which forms the foundation for modern algebraic geometry.

An important component of the course will be solving exercises from Hartshorne and other sources. Students are expected to engage actively with problems, as this is essential for mastering the techniques used throughout the subject.

### Tentative Topics (subject to adjustment)

- Affine varieties and the Zariski topology

- Projective varieties and projective space
- Regular functions and morphisms of varieties
- Rational maps and birational equivalence
- Nonsingular varieties and the Jacobian criterion
- Nonsingular curves
- Divisors and basic intersection theory (time permitting)
- Hilbert polynomials and Bézout's theorem (time permitting)
- Introduction to sheaves
- Locally ringed spaces
- Affine schemes and basic properties of schemes

**Textbooks and Course Notes:**

Primary reference:

- *Algebraic Geometry* by Robin Hartshorne.

Additional course notes will be provided. These notes will draw from Hartshorne's text and selected lecture notes by Takumi Maruyama.

**Additional References:**

- *Basic Algebraic Geometry, Part I* by I. R. Shafarevich
- *Introduction to Commutative Algebra* by M. Atiyah and I. Macdonald
- *Commutative Algebra with a View Toward Algebraic Geometry* by D. Eisenbud

**MA 57100 Elementary Topology**

**Instructor:** Professor Jeremy Miller

**Course Number:** MA 57100

**Credits:** Three

**Meeting Time:** 9:30–10:20 AM Monday / Wednesday / Friday

## Description

The course covers point-set topology and the fundamental group. Topics in point-set topology include compactness, connectedness, separation axioms, and metric spaces. Topics related to the fundamental group include covering spaces, Seifert–Van Kampen theorem, and abelianization.

## Textbook

We will use *Topology; a first course* by James Munkres.

## MA 57400 Numerical Optimization

**Instructor:** Professor Xiangxiong Zhang

**Course Number:** MA 57400

**Credits:** Three

**Meeting Time:** 10:30–11:20 AM Monday / Wednesday / Friday

## Description

This graduate-level course focuses on the development and rigorous analysis of practical numerical optimization algorithms that arise in contemporary large-scale machine learning and data science. A central theme of the course is convergence analysis, with particular emphasis on tools from monotone operator theory, fixed-point iterations, duality, and operator splitting methods. The course is organized into four parts (approximately 3–4 weeks each):

### Part I: Smooth Optimization

Convergence and convergence rate analysis of gradient descent, Nesterov’s accelerated gradient method, Newton and quasi-Newton methods, and conjugate gradient methods under standard smoothness and convexity assumptions.

### Part II: Nonsmooth Optimization

Convergence analysis of the subgradient method, proximal gradient methods, and accelerated proximal gradient methods. First-order splitting methods including PDHG (Primal–Dual Hybrid Gradient), ADMM (Alternating Direction Method of Multipliers), and Douglas–Rachford splitting. Selected topics in nonconvex optimization will also be discussed.

### Part III: Stochastic Algorithms

Convergence theory for randomized coordinate descent, and stochastic gradient descent.

### Part IV: Riemannian Optimization

A brief introduction to optimization algorithms on matrix manifolds, including Riemannian gradient descent and Riemannian conjugate gradient methods.

### Prerequisites

MA 511 and MA 504 (or equivalent background).

### References

- Amir Beck, *Introduction to Nonlinear Optimization*.
- Ernest Ryu and Wotao Yin, *Large-Scale Convex Optimization: Algorithms and Analyses via Monotone Operators*.
- P.-A. Absil, R. Mahony, and R. Sepulchre, *Optimization Algorithms on Matrix Manifolds*.

Past lecture notes from Fall 2024 are available at:

<https://www.math.purdue.edu/~zhan1966/teaching/574/2024/index.html>

Compared to Fall 2024, the Fall 2026 offering will place less emphasis on operator splitting methods and devote more time to modern techniques for analyzing nonconvex optimization problems, including the Kurdyka–Łojasiewicz (KL) inequality.

## MA 58400 Algebraic Number Theory

**Instructor:** Professor Ilia Shkredov

**Course Number:** MA 58400

**Credits:** Three

**Meeting Time:** 12:30–1:20 PM Monday / Wednesday / Friday

### Catalog Description

Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and in-

ertia groups, completions and local fields.

## **MA 58500 Mathematical Logic I**

**Instructor:** Professor Margaret Thomas

**Course Number:** MA 58500

**Credits:** Three

**Meeting Time:** 10:30–11:20 AM Monday / Wednesday / Friday

### **Description**

A first course in mathematical logic, oriented towards the branch of logic called model theory (this is the study of mathematical structures in terms of their logical properties, with many applications across and beyond mathematics, including to algebraic geometry, group theory, number theory, combinatorics, analytic geometry, operator algebras, dynamical systems, machine learning, ...). The goal is to introduce central ideas and theorems of mathematical logic and set theory, and students are encouraged to take this course if they are either interested in possible research involving model theory or keen to learn a perspective from logic that might be applicable to other areas of mathematics.

The course will start with an introduction to predicate calculus (first-order logic), covering languages, structures, theories, formal proof, Gödel's Completeness Theorem and the Compactness Theorem of first-order logic. This will also require a diversion into axiomatic set theory (ZF axioms, the Axiom of Choice, ordinals and cardinals) in order to illuminate the power of certain model-theoretic ideas, some of which will be illustrated by considering questions of (finite) axiomatizability for certain natural classes of mathematical objects. Thereafter, the focus will shift to further topics from the foundations of mathematical logic, in particular the formalization of elementary number theory and the beginnings of computability theory (including primitive recursion), which lead to the Gödel Incompleteness Theorems and the undecidability of arithmetic (however, given the intended focus of this course, these might not be covered in such great depth here).

### **Prerequisites**

Courses in abstract algebra and real analysis would be advisable (preferably at the graduate level), as subject matter from these areas will form the basis of illustrative examples. There is no requirement to have taken a course in logic previously.

## **Assessment**

Problem sets will be offered regularly (approximately once every two weeks) throughout the semester. The expectation is that students would demonstrate engagement with the course by submitting written solutions to a reasonable subset of these problems and engaging actively in class discussions. There will also be a midterm and a final examination.

## **Textbook**

The course will be based on material from a variety of sources, but the text that we will follow most closely is Marker, D., *An Invitation to Mathematical Logic*, Graduate Texts in Mathematics, vol. 301, Springer, New York, 2024 (available as an e-book through Purdue Libraries).

## **MA 59500CTG Topological Methods in Group Theory**

**Instructor:** Professor Yash Lodha

**Course Number:** MA 59500CTG

**Credits:** Three

**Meeting Time:** 4:30–5:45 PM Tuesday / Thursday

## **Description**

We will study infinite discrete groups using topological and cohomological methods. We will learn about Cayley 2-complexes, Eilenberg–MacLane complexes, finiteness properties, cohomology of groups and notions of dimension. I will review some basic covering space theory at the beginning of the semester, but some knowledge of basic algebraic topology will be assumed. We will use the textbook “Topological methods in group theory” by Ross Geoghegan as a guiding text, and other supplemental texts and research articles. We will also learn about some major open problems in the area such as the Whitehead asphericity conjecture and the Eilenberg–Ganea conjecture. The major goal will be to help students learn some cutting edge tools to work on open problems in group theory.

## **MA 59500EC Elliptic Curves**

**Instructor:** Professor Daniel Tuan-Dan Le

**Course Number:** MA 59500EC

**Credits:** Three

**Meeting Time:** 9:00–10:15 AM Tuesday / Thursday

### **Description**

Elliptic curves are the simplest “non-trivial” objects across a wide swath of subjects including complex and algebraic geometry and number theory. They correspondingly played a large role in the development of all of these. In the course, elliptic curves will serve as an introduction to general phenomena in these areas. The course will start with analytic aspects before moving to algebraic and arithmetic aspects. Our focus will be on (co)homology of elliptic curves in various guises.

### **Prerequisites**

Prerequisites are complex analysis and field theory. Some knowledge of commutative algebra, algebraic number theory, and algebraic varieties is recommended.

## **MA 59500HA Homological Algebra**

**Instructor:** Professor Saugata Basu

**Course Number:** MA 59500HA

**Credits:** Three

**Meeting Time:** 10:30–11:20 AM Monday / Wednesday / Friday

### **Description**

Course description to be announced.

## **MA 59500QI Mathematics for Quantum Information Science**

**Instructor:** Professor Eric Samperton

**Course Number:** MA 59500QI

**Credits:** Three

**Meeting Time:** 10:30–11:45 AM Tuesday / Thursday

## Description

This course introduces mathematics that is useful for various aspects of quantum information science, divided into four parts. The first consists of a rigorous mathematical treatment of quantum mechanics, quantum states, and quantum channels; highlights may include the no-cloning theorem, Holevo's theorem and Bell violations. The second part introduces various approaches to quantum computing, with most time spent on quantum circuits—particularly the existence of finite universal gate sets and the Solovay–Kitaev theorem. Additional topics may include adiabatic quantum computing, measurement-based quantum computing, and topological quantum computing. Part three consists of a brief introduction to quantum error correction, including the stabilizer formalism, the Gottesman–Knill theorem, and the threshold theorem. The final part introduces the roles of symmetry and topology in quantum systems, and may include topics such as the analysis of spin coupling via representation theory and the use of tensor categories to describe the operations performed on a topological quantum computer.

## MA 59500SML Stochastic Calculus Methods in Machine Learning

**Instructor:** Professor Samy Tindel

**Course Number:** MA 59500SML

**Credits:** Three

**Meeting Time:** 3:00–4:15 PM Tuesday / Thursday

## Description

In recent years, machine learning research has been an avid consumer of advanced methods in stochastic calculus and stochastic analysis. The aim of this course is to lay out the foundations and explore recent applications of these tools. Among the topics we intend to cover are:

1. Reversed diffusion processes and generative AI algorithms
2. Stochastic control and neural networks
3. Rough paths and the signature method for machine learning
4. Relaxed control and reinforcement learning

We will first review the mathematical background required to fully grasp each of those themes. The evaluation of the students will be based on reading and presenting recent papers.

## References

There is no real textbook covering the material of the course. The basic stochastic calculus tools will mostly be taken from Karatzas–Shreve’s book *Brownian motion and stochastic calculus*. A good introduction to stochastic control can be found in Yong and Zhou’s book *Stochastic controls*. For an introduction to rough paths, I will follow some lecture notes by Xi Geng (available on his webpage). Then for most applications, we will rely on recent articles.

## Prerequisites

MA 532 and MA 538. Enforced for undergraduate students.

## MA 59500SQM Semiclassical Analysis in Quantum Mechanics

**Instructor:** Professor Kiril Datchev

**Course Number:** MA 59500SQM

**Credits:** Three

**Meeting Time:** 3:30–4:20 PM Monday / Wednesday / Friday

## Description

Semiclassical analysis is the mathematical study of wave-particle correspondence. The simplest example is a light or sound wave, which at scales much larger than the wavelength (the familiar setting for light and sound) travels along the same straight lines as any point particle. With the advent of quantum mechanics, this correspondence was shown to apply in varying degrees to all matter, with salient features of complicated wave behavior explained in simpler particle terms.

This course will begin with the first general semiclassical approximation, the WKB approximation. Our focus will be the fundamental problem of computing spectra (usually eigenvalues) and the associated states (eigenvectors). Under broad conditions these are governed by universal asymptotics, which go back to work of Rayleigh on acoustics and black body radiation, and have appeared in various guises in Weyl’s law for eigenvalues of the Laplacian, in Thomas and Fermi’s theory of electronic structure, and in the far-reaching generalizations and refinements these have continued to see.

To understand these, we will develop technical tools, including pseudodifferential operators, functional calculus, and trace formulas. To some extent we will follow the textbooks ‘Spectral Asymptotics in the Semi-Classical Limit’ by Dimassi and Sjöstrand, and ‘Semiclassical Analysis’ by Zworski, as well as the online course notes ‘An Introduction to Semiclassical

Analysis' by Nonnenmacher, but our approach will be more elementary, and we will not go as far as those references do.

We will then study recent results on the contribution of boundaries to spectral asymptotics for the Schrödinger equation. Asymptotic methods are important here because this equation becomes too complicated to be solved directly, even numerically, as the number of particles grows. Our asymptotics make it possible to compute energies accurately and efficiently, yielding improvements to approximations used in density functional theory.

### **Prerequisites**

Graduate level functional analysis or quantum mechanics. The intended audience includes students in either mathematics or physics who have an interest in (but not necessarily background in) the other subject.

## **MA 61100 Methods of Applied Mathematics**

**Instructor:** Professor Isaac Harris

**Course Number:** MA 61100

**Credits:** Three

**Meeting Time:** 4:30–5:45 PM Tuesday / Thursday

### **Description**

Banach and Hilbert spaces; linear operators; spectral theory of compact linear operators; applications to linear integral equations and to regular Sturm-Liouville problems for ordinary differential equations. This course will focus on the application of Functional Analysis. In particular, we will study applications to Integral Equations, (Partial) Differential Equations, as well as Inverse Problems.

### **Reference Texts**

- Erwin Kreyszig, *Introductory Functional Analysis with Applications*.
- Sandro Salsa, *Partial Differential Equations in Action: From Modeling to Theory*.

# MA 69700GTT Geometry and Topology of 3-Manifolds

**Instructor:** Professor Lyuzhou Chen

**Course Number:** MA 69700GTT

**Credits:** Three

**Meeting Time:** 10:30–11:45 AM Tuesday / Thursday

## Description

Manifolds in dimension three form a rich family that is at the time comprehensible (unlike the higher dimensional cases). Dimension three is just above the dimension (two) for which we feel easy to visualize. It is also the special dimension where geometry is as rich as topology. The goal of this course is to introduce the basic/classical theory of 3-manifolds from a more modern point of view.

We plan to cover basic examples and constructions (Heegaard splitting, link complements, Dehn surgery, surface bundles), various major (more classical) theorems that lead to the notions of the prime decomposition, the JSJ decomposition, and the Thurston norm. We will also briefly discuss some more recent breakthroughs (with few detailed proofs): The geometrization theorem (which implies the 3-d Poincaré conjecture), the surface subgroup theorem, and the virtual fibering and virtual Haken theorems.

## Prerequisites

We assume some basic knowledge in algebraic topology, differential topology/geometry, manifolds, and group actions.