Numerical Analysis
Instructor: Professor Steven Dong
Course Number: MA 51400
Credits: Three
Time: 8:30–9:20 AM MWF

Catalog Description

(CS 51400) Iterative methods for solving nonlinear; linear difference equations, applications to solution of polynomial equations; differentiation and integration formulas; numerical solution of ordinary differential equations; roundoff error bounds.

Introduction To Probability
Instructor: Professor Gregery Buzzard
Course Number: MA 51900
Credits: Three
Time: 12:30–1:20 PM MWF

Catalog Description

(STAT 51900) Algebra of sets, sample spaces, combinatorial problems, independence, random variables, distribution functions, moment generating functions, special continuous and discrete distributions, distribution of a function of a random variable, limit theorems.

Introduction To Partial Differential Equations
Instructor: Professor Arshak Petrosyan
Course Number: MA 52300
Credits: Three
Time: 10:30–11:45 AM TTh

Catalog Description

First order quasi-linear equations and their applications to physical and social sciences; the Cauchy-Kovalevsky theorem; characteristics, classification and canonical forms of linear equations; equations of mathematical physics; study of Laplace, wave and heat equations; methods of solution.

Probability Theory II
Instructor: Professor Rodrigo Banuelos
Course Number: MA 53900
Credits: Three
Time: 10:30–11:20 AM MWF

Catalog Description

(STAT 53900) Convergence of probability laws; characteristic functions; convergence to the normal law; infinitely divisible and stable laws; Brownian motion and the invariance principle.

Real Analysis And Measure Theory
Instructor: Professor Jonathon Peterson
Course Number: MA 54400
Credits: Three
Time: 9:30–10:20 AM MWF

Catalog Description

Metric space topology; continuity, convergence; equicontinuity; compactness; bounded variation, Helly selection theorem; Riemann-Stieltjes integral; Lebesgue measure; abstract measure spaces; LP-spaces; Holder and Minkowski inequalities; Riesz-Fischer theorem.

Introduction To Abstract Algebra
Instructor: Professor Freydoon Shahidi
Course Number: MA 55300
Credits: Three
Time: 10:30–11:20 AM   MWF

Catalog Description

Group theory: Sylow theorems, Jordan-Holder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

Linear Algebra
Instructor: Professor Saugata Basu
Course Number: MA 55400
Credits: Three
Time: 12:30–1:20 PM   MWF

Catalog Description

Review of basics: vector spaces, dimension, linear maps, matrices determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

Abstract Algebra I
Instructor: Professor Giulio Caviglia
Course Number: MA 55700
Credits: Three
Time: 12:30–1:20 PM   MWF

Catalog Description

Review of fundamental structures of algebra (groups, rings, fields, modules, algebras); Jordan-Holder and Sylow theorems; Galois theory; bilinear forms; modules over principal ideal domains; Artinian rings and semisimple modules. Polynomial and power series rings; Noetherian rings and modules; localization; integral dependence; rudiments of algebraic geometry and algebraic number theory; ramification theory.

Introduction To Differential Geometry And Topology

Instructor: Professor Manuel Rivera
Course Number: MA 56200
Credits: Three
Time: 10:30–11:45 AM TTh

Description

This course will be an introduction to the geometry and topology of manifolds. We will begin by defining manifolds and smooth maps, derivatives and tangent spaces, and prove the inverse and implicit function theorems. We will continue by discussing transversality and intersection on manifolds. Then we will discuss vector fields, differential forms, and integration on manifolds. Some of the highlights of the course include the Whitney embedding theorem, Poincaré-Hopf index theorem, Stokes' theorem, and Gauss-Bonnet theorem. We will follow Guillemin and Pollack's Differential Topology book, supplemented by Lee's Introduction to Smooth Manifolds and Milnor's Topology from the Differentiable Viewpoint. The course will be accessible to any students who has basic knowledge of analysis, linear algebra, and familiarity with basic topological concepts in Euclidean space.

Elementary Topology

Instructor: Professor Ralph Kaufmann
Course Number: MA 57100
Credits: Three

Time: 8:30–9:20 AM   MWF

Catalog Description

Fundamentals of point set topology with a brief introduction to the fundamental group and related topics, topological and metric spaces, compactness, connectedness, separation properties, local compactness, introduction to function spaces, basic notions involving deformations of continuous paths.

Algebraic Number Theory
Instructor: Professor Daniel Le
Course Number: MA 58400
Credits: Three
Time: 2:30–3:20 PM   MWF

Catalog Description

Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.

Introduction To Algebraic Geometry
Instructor: Professor Jaroslaw Wlodarczyk
Course Number: MA 59500A
Credits: Three
Time: 10:30–11:45 PM   TTh

Description

We shall give an introductory course in Algebraic Geometry based upon Hartshorne's Algebraic Geometry: Chapters 1, 2. The important part of the class will be solving problems from Hartshorne's book and others. The emphasis of this course is on proofs and algebraic tools used in Algebraic Geometry.

The tentative list of the topics includes but is not limited to: Affine and Projective varieties, Morphisms of varieties, Rational maps, Non-singular varieties, Sheaves, Schemes, Separated and proper morphisms, Normal varieties,Coherent and Quasicoherent modules.

Text: *Algebraic geometry* (AG) by Hartshorne. *Vakil's notes* (V)

Additional reading: *Balgebraic Geometry* (Part 1) by Shafarevich, *Introduction to commutative algebra* by Atiyah and Macdonald, *Commutative algebra with a view towards algebraic geometry* by Eisenbud.

Homological Stability
Instructor: Professor Jeremy Miller
Course Number: MA 59500G
Credits: Three
Time: 12:00–1:15 PM TTh

Description

Given a sequence of spaces X_n, homological stability is a pattern where the ith homology group $H_i(X_n)$ does not depend on i for i sufficiently large compared with n. Examples of this include configuration spaces of points in a manifold, moduli spaces of curves, and classifying spaces of groups such as general linear groups and symmetric groups. The course will begin with background and motivation. We will briefly cover the theory of fiber bundles, fibrations, classifying spaces, simplicial sets, loop spaces, spectral sequences, etc. Then we will develop techniques for proving homological stability and computing the stable homology. Time permitting, we will discuss an operatic approach to proving homological stability.

Harmonic Analysis & Applications
Instructor: Professor Victor Lie
Course Number: MA 59500H
Credits: Three
Time: 12:00–1:15 PM   TTh

Description

In this course we will present an array of fundamental concepts, starting with basic Calderon-Zygmund theory, Hardy-Littlewood maximal function, real (Marcinkiewicz) and complex interpolation, elements of functional analysis (open mapping, closed graph and Hahn-Banach theorems) with applications in Fourier Series, Fourier Transform, elements of restriction theory with applications in PDE (local wellposedness for some fundamental PDE and Strichartz estimates), Kakeya problem over finite fields etc. A good reference for this class would be the Fourier Series and Functional Analysis books by Stein and Shakarchi but the material stated above would only partly overlap. Other useful materials include Fourier Analysis lecture notes of Terry Tao and slides prepared by me over the years.

If time allows us/there is interest we can discuss several other more advanced topics, such as: the celebrated thorem of Carleson on the pointwise convergence of Fourier Series, the boundedness of the Bilinear Hilbert Transform or the recent work on the so-called LGC methodology.

Lie Algebras
Instructor: Professor Oleksandr Tsymbaliuk
Course Number: MA 59500L
Credits: Three
Time: 3:00–4:15 PM   TTh

Description

This is an introductory course on Lie algebras. Our main focus will be the study of finite-dimensional Lie algebras, with the key emphasis placed on the semisimple ones that do admit a beautiful complete theory. The course is expected to fit a wide range of students: both graduate and strong undergraduate mathematics students, as well as graduate physics students.

A Lie algebra is a vector space equipped with a bilinear operation, called a *Lie bracket*. Despite this abstract definition, one should not forget their historical origin in the context of the Lie group theory – a mathematical treatment of continuous symmetries. Notably, Lie groups are determined by their linear approximation at the identity, called *the Lie algebra of a Lie group*. This allows to reformulate the above theory in purely algebraic terms of Lie algebras (viewing them as spaces of "infinitesimal" symmetries) and motivates many related problems.

While being of independent interest, this subject finds interesting applications in other areas of mathematics and mathematical physics: algebraic combinatorics, differential geometry, topology, number theory, partial differential equations, quantum physics, and many more.

Tentative list of topics: Lie groups and the exponential map, nilpotent and solvable Lie algebras, theorems of Engel and Lie, Cartan subalgebras, Killing form and Cartan criteria, structure of semisimple Lie algebras, root systems, Weyl group, Dynkin diagrams, classification and construction of semisimple Lie algebras, universal enveloping algebras and the Poincaré-Birkhoff-Witt theorem, representation theory of semisimple Lie algebras, Weyl character formula, theorems of Levi and Maltsev, Harish-Chandra isomorphism.

Prerequisites: Basic notions from algebra (especially linear algebra).

Neural Networks and Numerical PDEs

Instructor: Professor Zhiqiang Cai
Course Number: MA 59500N
Credits: Three
Time: 10:30–11:45 AM TTh

Description

Neural Networks (NNs) have achieved astonishing performance in computer vision, natural language processing, and many other artificial intelligence tasks. This success encourages wide applications to other fields, including recent studies of using NNs to numerically solve partial differential equations (PDEs).

This course will first introduce NNs as a new class of approximating functions and its approximation properties to functions. We will then discuss NN-based numerical methods for solving computationally challenging problems arising from continuum mechanics with applications to Darcy's flow in porous media, elastic equations for solids, incompressible Newtonian fluid flow, and Maxwell's equations in electromagnetic. Topics include accurate discretization method, efficient training algorithm, and adaptive construction of NN.

A tentative list of contents:

1. Mathematical Models of Continuum Mechanics
2. Least-squares Formulations
3. Neural Networks and their Approximation Properties
4. Least–Squares Neural Network Methods
5. Training Algorithms
6. Adaptive Network Enhancement Method

Prerequisite: MA 514 or equivalent

Numerical Solutions For Ordinary Differential Equations
Instructor: Professor Di Qi
Course Number: MA 59500ODE (573000)
Credits: Three
Time: 2:30–3:20 PM MWF

Description

This course will cover the fundamentals and applications of numerical methods essential for solving differential equations. Solutions of ordinary differential equations are discussed, including single and multistep methods for initial value problems, Runge-Kutta schemes, convergence and stability, and methods for stiff problems such as exponential temporal integrators and multigrid iterative solvers. Numerical solutions to stochastic differential equations and Monte Carlo methods will also be discussed. The course will continue to applications of the time stepping schemes in solving boundary and eigenvalue problems of partial differential equations.

Prerequisites: MA 51400 or CS 51400

Commutative Algebra

Instructor: Professor William Heinzer
Course Number: MA 65000
Credits: Three
Time: 3:30–4:20 PM   MWF

Description

In this course I hope to present selected topics from the book "Integral Domains Inside Noetherian Power Series Rings: Constructions and Examples" written by Christel Rotthaus, Sylvia Wiegand, and me. There will be no exams and no graded HW in the course. Students will be encouraged to participate actively in class discussion.

For the topics to be considered in the course, two motivating questions are:

Question 1. What rings lie between a Noetherian integral domain and its field of fractions?

Question 2. Let I be an ideal of a Noetherian integral domain R and let R^* be the I-adic completion of R. What rings lie between R and R^*?

For example, if x and y are indeterminates over a field k, R is the polynomial ring $k[x, y]$, and I is the principal ideal xR, what rings lie between R and the mixed polynomial-power series ring $R^* = k[y][[x]]$?

Matrix Methods for Data Science
Instructor: Professor Jianlin Xia
Course Number: MA 69200MM
Credits: Three
Time: 1:30–2:45 PM TTh

Description

Matrix methods play a key computational role in modern data analysis, scientific computing, and engineering simulations. The course will cover some useful matrix methods for several data science topics. We will focus on fast and efficient matrix computations that can benefit machine learning, data analysis, and also other numerical computations. Selected topics include:

1. Matrix models of neural networks and machine learning.
2. Data matrices and matrix decompositions.
3. Randomized data probing, compression, and approximation.
4. Stochastic/randomized solvers for linear algebra and optimization.
5. Kernel matrix methods and fast transformations.
6. Fast multipole methods and structured matrices.
7. Fast direct solvers and eigenvalue solvers with applications to data analysis.

Knowledge in basic numerical analysis and linear algebra is strongly suggested. There will be no comprehensive final exam. Reference books/papers, lecture notes, test codes, and other resources will be provided.