MA 16100
Final Exam - Fall 2024
12/11/2024
TEST/QUIZ NUMBER:

41

NAME ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯ YOUR TA'S NAME ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

STUDENT ID # ⎯⎯⎯⎯⎯⎯⎯⎯⎯ RECITATION TIME ⎯⎯⎯⎯⎯⎯⎯⎯⎯

You must use a #2 pencil on the scantron answer sheet. Fill in the following on your scantron and blacken the bubbles

1. Your name. If there aren't enough spaces for your name, fill in as much as you can.

2. Section number with a leading zero, e.g. **0032**. (If you don't know your section number, ask your TA.)

3. Test/Quiz number: **41**

4. Student Identification Number: **This is your Purdue ID number with two leading zeros**

5. Blacken in your choice of the correct answer on the scantron answer sheet for questions 1–20.

There are **20** questions, each worth 5 points, for a total of 100 points. Do all your work in this exam booklet. Use the back of the test pages for scrap paper. Turn in both the scantron and the exam booklet when you are finished.

You may not leave the room before 7:20PM. If you finish the exam between 7:20PM and 8:50PM, you may leave the room after turning in the scantron sheet and the exam booklet. If you don't finish before 8:50PM, you <u>must remain seated</u> until your TA comes and collects your scantron sheet and your exam booklet.
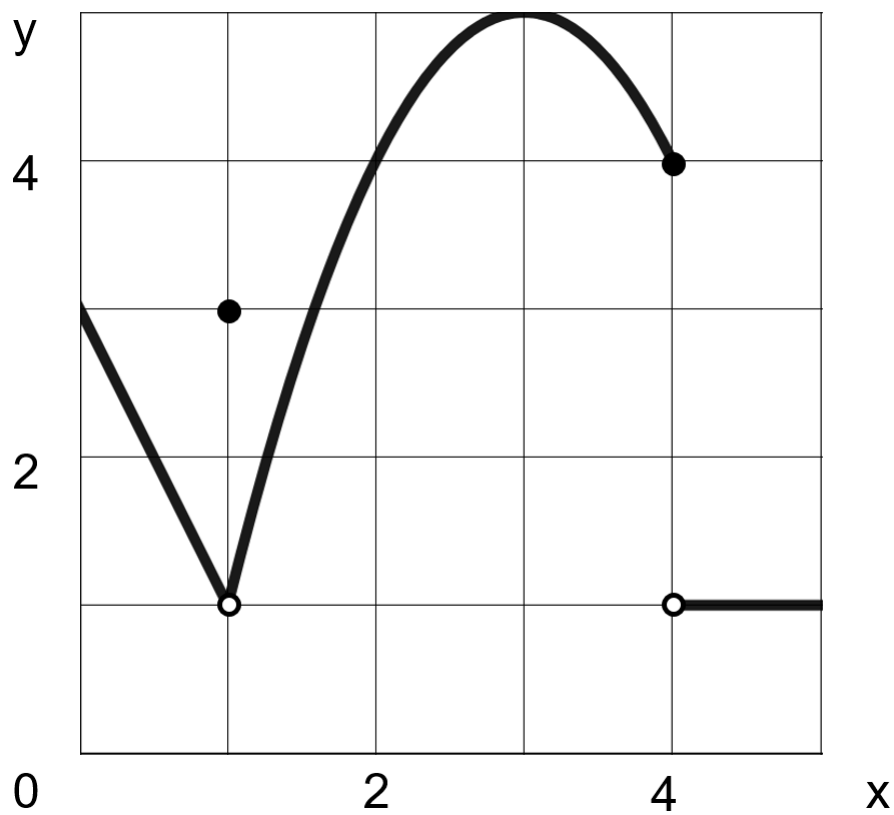
EXAM POLICIES

1. Students may not open the exam booklet until instructed to do so.

2. Students must obey the orders and requests by all proctors, TAs, and lecturers.

3. No student may leave in the first 20 min or in the last 10 min of the exam.

4. Books, notes, calculators, or any electronic devices are not allowed on the exam, and they should not even be in sight in the exam room. Students may not look at anybody else's test, and may not communicate with anybody else except, if they have a question, with their TA or lecturer.

5. After time is called, students must put down all writing instruments and remain in their seats, while the TAs will collect the scantrons and the exams.

6. Any violation of these rules and any act of academic dishonesty may result in severe penalties. Additionally, all violators will be reported to the Office of the Dean of Students.

I have read and understand the exam rules stated above:

STUDENT SIGNATURE: ⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯⎯

1. For the function graphed below, which of the following statements is TRUE?

A. $\lim\limits_{x\to 1} f(x)$ does not exist.

B. f is discontinuous at $x = 1$.

C. $\lim\limits_{x\to 4} f(x)$ exists.

D. f is differentiable at $x = 1$.

E. $\lim\limits_{x\to 4^-} f(x) = 1$.

2. Given $g(x) = \displaystyle\int_{10}^{\sin(x)} \sqrt{t^4 + 1} \; dt$, find $g'(x)$.

  A. $4\cos^3(x)\sqrt{\sin^4(x) + 1}$

  B. $\sqrt{x^4 \sin(x) + 1}$

  C. $\cos(x)\sqrt{\sin^4(x) + 1}$

  D. $4x^3\sqrt{x^4 \sin(x) + 1}$

  E. $\sqrt{\sin^4(x) + 1}$

3. Find the derivative $\dfrac{dy}{dx}$ of $y = x^{x^3}$.

    A. $y' = x^{x^3} \cdot (1 + 3\ln(x))$

    B. $y' = x^{x^3-1} \cdot (x^2 + 3x^2 \ln(x))$

    C. $y' = x^{x^3} \cdot (x^2 + 3x^2 \ln(x))$

    D. $y' = x^3 \cdot x^{x^3-1} \cdot (3x^2)$

    E. $y' = x^{x^3-1} \cdot (3x^2)$

4. Determine $y'(1)$ for $y = \ln(5x) + \arctan(3x)$.

A. $\dfrac{53}{10}$

B. $\dfrac{3}{10}$

C. $\dfrac{1}{2}$

D. None of the other answers

E. $\dfrac{11}{10}$

5. The derivative of a function f is given by $f'(x) = (x-2)^2(x-3)^3(x-4)$. Which of the following are correct?

I. $f(3)$ is a local maximum and $f(4)$ is a local minimum of $f(x)$.

II. $f(x)$ is increasing on the interval $(2,4)$.

III. $f(x)$ is decreasing on $(-\infty, 2)$ and increasing on $(2, \infty)$.

    A. I and III only

    B. I only

    C. II only

    D. II and III only

    E. III only

6. Given that $F(x) = \int_0^x f(t)\, dt$. Which of the following statements are true:

I. $F(x)$ is increasing when $f(x) < 0$.

II. $F(x)$ has a critical point when $f(x) = 0$.

III. $F(x)$ is concave up when $f(x)$ is increasing.

    A. II only

    B. I and II only

    C. None of them are true

    D. II and III only

    E. I, II and III

7. Suppose the range of $f(x)$ is $[-1, 3]$. If

$$g(x) = 2f(3x - 1) - 5$$

then what is the range of the function $g(x)$?

A. $[-2, 10]$
B. $[1, 7]$
C. $[3, 11]$
D. $[-7, 1]$
E. $[-1, 3]$

8. Find the correct formula that approximates the area under the curve $f(x) = 49 - x^2$ on the interval $[0, 7]$ using right-endpoint Riemann sums with n rectangles.

A. $\displaystyle\sum_{i=1}^{n} \left(49 - \left(\frac{7i}{n}\right)^2\right) \cdot \frac{7}{n}$

B. $\displaystyle\sum_{i=1}^{n} \left(49 - \left(\frac{7(i-1)}{n}\right)^2\right) \cdot \frac{7}{n}$

C. $\displaystyle\sum_{i=1}^{n} \left(49 - \left(\frac{7i}{n}\right)^2\right) \cdot 7$

D. $\displaystyle\sum_{i=0}^{n-1} \left(49 - \left(\frac{7i}{n}\right)^2\right) \cdot \frac{7}{n}$

E. $\displaystyle\sum_{i=1}^{n} \left(49 - i^2\right) \cdot \frac{7}{n}$

9. Evaluate the limit:

$$\lim_{h \to 0} \frac{\sin(2x)\cos(2h) + \cos(2x)\sin(2h) - \sin(2x)}{h}$$

A. $2\sin(2x) + 2\cos(2x)$

B. $\cos(2x)$

C. $2\cos(2x) + \sin(2x)$

D. $2\cos(2x)$

E. $2\sin(2x)$

10. Suppose that $\int_1^9 f(x)\,dx = -1$, $\int_1^9 h(x)\,dx = 3$, $\int_7^9 f(x)\,dx = 5$, and $\int_7^9 h(x)\,dx = 4$.

Evaluate $\int_1^7 (2f(x) - 3h(x))\,dx$.

    A. -2

    B. -15

    C. -9

    D. None of the other answers

    E. -11

11. A sample radioactive element initially has mass of 24g. After 2 minutes the sample of that element has mass 2g. At what time (in minutes after the initial time) is the mass equal to 4g?

A. $\dfrac{2\ln 6}{\ln 12}$

B. $\dfrac{3\ln 8}{\ln 3}$

C. $\dfrac{2\ln 12}{\ln 6}$

D. $\dfrac{3\ln 6}{2\ln 2}$
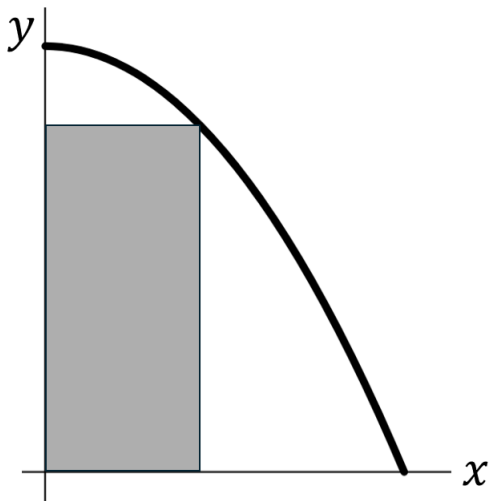
E. $\dfrac{\ln 6}{\ln 12}$

12. Evaluate:

$$\lim_{x\to 0} \frac{5x}{\sqrt{36x^2+81}-9}$$

A. $-\infty$

B. The limit does not exist

C. 0

D. $\dfrac{5}{6}$

E. ∞

13. An empty cylindrical water tank with a radius of 7 feet is being filled with water, but at the same time, it has a small leak at the bottom. Water is being added to the tank at a rate of 6 cubic feet per minute. However, due to the leak, water is also draining out at a rate of 4 cubic feet per minute. How fast is the water level rising when the water is 3 feet deep?
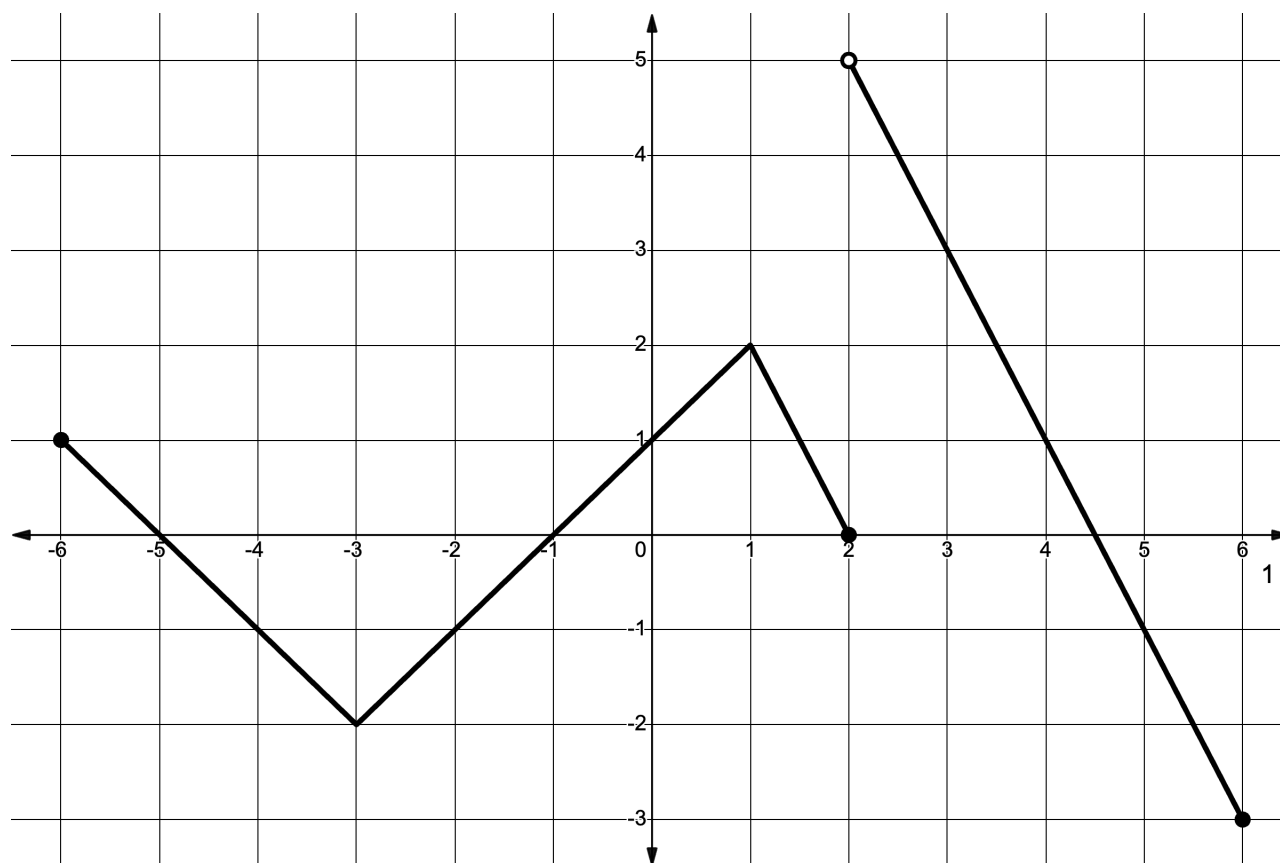
A. $\dfrac{2}{49\pi}$ ft/min

B. $\dfrac{2}{7\pi}$ ft/min

C. $\dfrac{7}{6\pi}$ ft/min

D. $\dfrac{6}{49\pi}$ ft/min

E. $\dfrac{4}{49\pi}$ ft/min

14. A rectangle in the first quadrant is constructed with its base on the x-axis, one side along the y-axis, and it's top right vertex on the parabola $y = 48 - x^2$. What is the height of the rectangle that maximizes the area of the rectangle?

A. $y = 12$

B. $y = 24$

C. $y = 36$

D. $y = 16$

E. $y = 32$

15. For the function graphed below, determine the values of the two stated integrals.

A. $\int_{-5}^{-1} f(x)\ dx = -4$ and $\int_{4}^{2} f(x)\ dx = -6$

B. $\int_{-5}^{-1} f(x)\ dx = -4$ and $\int_{4}^{2} f(x)\ dx = 6$

C. $\int_{-5}^{-1} f(x)\ dx = 4$ and $\int_{4}^{2} f(x)\ dx = 6$

D. None of the other answers

E. $\int_{-5}^{-1} f(x)\ dx = 4$ and $\int_{4}^{2} f(x)\ dx = -6$

16. Evaluate the following integral.

$$\int_1^3 \frac{x^3 - 4}{x}\, dx$$

A. $\dfrac{26}{3} - \ln 3$

B. 3

C. $\dfrac{26}{3} - \ln 12$

D. $\dfrac{2}{3}$

E. $\dfrac{26}{3} - 4\ln 3$

17. Find the values of a and b so that $f(x)$ defined below is continuous for all values of x.

$$f(x) = \begin{cases} x^3 + 2, & x < -1 \\ ax - b, & -1 \le x \le 2 \\ \dfrac{x^2 + 12}{x + 2}, & x > 2 \end{cases}$$

A. $a = 0, b = 4$

B. $a = 1, b = -2$

C. $a = 0, b = 1$

D. $a = 1, b = 2$

E. No value of a or b, since $\dfrac{x^2 + 12}{x + 2}$ is not continuous at $x = -2$

18. Determine the number of local extrema (max or min) of $f(x) = x - \cos x$ on the interval $(0, 4\pi)$.

   A. 1
   B. 4
   C. 0
   D. 2
   E. 3

19. Find the slope of the line tangent to the curve $x^4y - xy^3 = -2$ at the point $(-1, -1)$.

A. $-\dfrac{5}{2}$

B. $-\dfrac{5}{4}$

C. $-\dfrac{3}{4}$

D. $-\dfrac{2}{3}$

E. -1

20. Evaluate $\displaystyle\int_0^1 \frac{\tan^{-1}(x)}{x^2+1}\,dx$.

A. $\dfrac{\pi^2}{32}$

B. $\dfrac{\pi^2}{8}$

C. $\dfrac{\pi^2}{16}$

D. $\dfrac{1}{8}$

E. $\dfrac{1}{32}$

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