MA 16200: Second Midterm Examination

Fall 2024, Purdue University

Exam version: 01

Name: _____ PUID #: _____

Exam Instructions:

- Follow these instructions carefully. Failure to do so may result in your exam being invalidated and/or an academic integrity violation. All suspected violations of academic integrity will be reported to the Office of the Dean of Students.
- Mark the circle of your recitation section below. Write your name and PUID on the top of this cover page. **DO NOT WRITE ANYTHING ELSE** on this cover page.

	Sec	Time	TA Name
◯	206	7:30AM	Gage Bachmann
◯	109	7:30AM	Lance Daley
◯	904	7:30AM	Luca Mossman
◯	906	7:30AM	Michael Poole
◯	210	7:30AM	Ehan Shah
◯	208	8:30AM	Gage Bachmann
◯	111	8:30AM	Lance Daley
◯	908	8:30AM	Luca Mossman
◯	902	8:30AM	Michael Poole
◯	212	8:30AM	Ehan Shah
◯	224	9:30AM	Niveditha Nerella
◯	216	9:30AM	Claudia Phagan
◯	107	10:30AM	Otto Baier
◯	222	10:30AM	Niveditha Nerella

	Sec	Time	TA Name
◯	214	10:30AM	Claudia Phagan
◯	113	11:30AM	Tausif Ahmed
◯	105	11:30AM	Otto Baier
◯	115	12:30PM	Tausif Ahmed
◯	101	12:30PM	Alexis Cruz Castillo
◯	103	1:30PM	Alexis Cruz Castillo
◯	218	1:30PM	Leo Shen
◯	220	2:30PM	Leo Shen
◯	117	3:30PM	Tifany Burnett
◯	204	3:30PM	Mohamad Mousa
◯	121	3:30PM	Juliet Raginsky
◯	119	4:30PM	Tifany Burnett
◯	202	4:30PM	Mohamad Mousa
◯	123	4:30PM	Juliet Raginsky

- This exam consists of 12 questions for a total of 100 points.
- You have exactly one hour to complete the exam.
- Do not open the exam booklet or start writing before the proctor signals the start of the exam.
- Write your PUID on every other page of the exam booklet. This will help us locate your test if the pages become separated. Only do this after the exam starts.
- Additional pages for scratch work can be found at the end of the booklet.
- Calculators, electronic devices, books, or notes are **NOT ALLOWED**.
- Students may not look at anybody else's exam, and may not communicate with anybody else except with their TA or instructor if there is a question.
- **DO NOT DETACH ANY PAGES** from the exam booklet.
- If you finish the exam before 8:55 pm, you may leave the room after turning in the exam booklet. You may not leave the room before 8:20 pm. If you don't finish before 8:55 pm, **YOU MUST REMAIN SEATED** until your TA comes and collects your exam booklet. You must stop working when the proctor signals the end of exam.

Good luck!

Multiple-choice Instructions:

- For multiple choice questions, fill the circles completely with a **#2 PENCIL** for your answer choices. If you need to change your answer choice, erase the mark completely.

DO: ○ ●

DON'T: ⊘ ⊗ ⊘ ⊘ ◉ ●

- Partial credit will not be awarded for multiple choice questions.

Fill-in-the-blank Instructions:

- For fill-in-the-blank questions, write your answers in the provided text boxes. Answers written entirely or partially outside of the boxes will not be graded.
- Only write the final answer in text boxes. Intermediate steps and scratch work should be completed in the blank space below each question.
- Write all your answers in one line. Fractions of the form $\frac{X}{Y}$ are okay to include.

DO: 10π $\frac{2x}{3} + \sin(x)$

DON'T: $5 \cdot 2\pi$ $\frac{2x}{3} + \sin(x)$ $\frac{2x}{3} - (-\sin(x)) = \frac{2x}{3} + \sin(x)$ $\frac{2x}{3}$ $+ \sin(x)$

- Partial credit will not be awarded for individual answer boxes. You may get partial credit for fill-in-the-blank questions if there are multiple text boxes in one question.

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1. (8 points) Which of the following statement(s) is/are TRUE?

   (I) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\lim_{n\to\infty} a_n \neq 0$.

   (II) If $\sum_{n=1}^{\infty} a_n$ is divergent, then $\sum_{n=43}^{\infty} a_n$ is divergent.

   (III) If $\lim_{n\to\infty} a_n$ does not exist, then $\sum_{n=1}^{\infty} a_n$ is divergent.

   ◯ (A) (I) only
   ◯ (B) (II) only
   ◯ (C) (III) only
   ◯ (D) All of (I), (II), and (III)
   ◯ (E) (II) and (III) only

2. (8 points) Express the repeated decimal $1.\overline{30} = 1.3030303030\ldots$ as a fraction.

   ◯ (A) $\frac{4}{3}$
   ◯ (B) $\frac{13}{10}$
   ◯ (C) $\frac{12}{11}$
   ◯ (D) $\frac{34}{33}$
   ◯ (E) $\frac{43}{33}$

3. (8 points) Evaluate

$$\int_1^\infty \frac{3x^2}{x^6+1}\,dx$$

- ◯ (A) 3
- ◯ (B) $\pi/2$
- ◯ (C) $\sqrt[3]{3}$
- ◯ (D) $\pi/4$
- ◯ (E) The integral does not converge.

4. (8 points) What is the correct form for the partial fraction decomposition of

$$\frac{x^3-1}{x^4+16x^2} \quad ?$$

- ◯ (A) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+4}$
- ◯ (B) $\dfrac{A}{x}+\dfrac{Bx+C}{x^2+16}$
- ◯ (C) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{Cx+D}{x^2+16}$
- ◯ (D) $\dfrac{A}{x}+\dfrac{B}{x^2}+\dfrac{C}{x+4}+\dfrac{D}{x-4}$
- ◯ (E) $\dfrac{A}{x}+\dfrac{B}{x+4}+\dfrac{C}{(x+4)^2}$

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5. (8 points) Evaluate

$$\int_0^4 \frac{2x}{x^2-4}\,dx$$

- ◯ (A) arctan(4)
- ◯ (B) ln(3)
- ◯ (C) 2/3
- ◯ (D) ln(8)
- ◯ (E) The integral does not converge.

6. (8 points) If

$$\frac{A}{x}+\frac{B}{x-1}+\frac{C}{(x-1)^2}=\frac{3x^2-x+1}{x^3-2x^2+x},$$

find the value of $A+B+C$.

- ◯ (A) 6
- ◯ (B) 1
- ◯ (C) 3
- ◯ (D) 2
- ◯ (E) 0

7. (8 points) For each one of the following three alternating series, is it absolutely convergent, conditionally convergent, or divergent?

$$S_1 = \sum_{k=1}^{\infty} \frac{(-1)^k}{\sqrt{k}}, \qquad S_2 = \sum_{k=1}^{\infty} \frac{(-1)^k k}{k+1}, \qquad S_3 = \sum_{k=1}^{\infty} \frac{(-1)^k}{2^k}.$$

- ◯ (A) S_1 is absolutely convergent; S_2 is divergent; S_3 is absolutely convergent.
- ◯ (B) S_1 is absolutely convergent; S_2 is divergent; S_3 is conditionally convergent.
- ◯ (C) S_1 is absolutely convergent; S_2 is conditionally convergent; S_3 is absolutely convergent.
- ◯ (D) S_1 is conditionally convergent; S_2 is divergent; S_3 is absolutely convergent.
- ◯ (E) S_1 is conditionally convergent; S_2 is divergent; S_3 is divergent.

8. (8 points) The sequence $\{a_n\}_{n=1}^{\infty}$ is defined by the recurrence relation

$$a_1 = 0, \quad \text{and} \quad a_{n+1} = (a_n)^2 - 2.$$

What is a_6?

- ◯ (A) -2
- ◯ (B) -62
- ◯ (C) 256
- ◯ (D) 0
- ◯ (E) 2

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9. (8 points) The series $\displaystyle\sum_{k=1}^{\infty} \frac{1}{\sqrt{k^2+k}}$ is

◯ (A) Divergent by the divergence test.

◯ (B) Divergent by the limit comparison test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$.

◯ (C) Convergent by the limit comparison test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$.

◯ (D) Divergent by the comparison test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k}$.

◯ (E) Convergent by the comparison test with $\displaystyle\sum_{k=1}^{\infty} \frac{1}{k^2}$.

10. (8 points) Which one of the following statements is TRUE?

◯ (A) If the sequence $\{a_n\}_{n=1}^{\infty}$ is divergent, then $\{1/a_n\}_{n=1}^{\infty}$ must be convergent.

◯ (B) If sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are divergent, then $\{a_n - b_n\}_{n=1}^{\infty}$ is also divergent.

◯ (C) If sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are divergent, then $\{a_n b_n\}_{n=1}^{\infty}$ is also divergent.

◯ (D) If sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent, then $\{a_n + b_n\}_{n=1}^{\infty}$ is also convergent.

◯ (E) If sequences $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are convergent, then $\{a_n/b_n\}_{n=1}^{\infty}$ is also convergent.

11. (8 points) This question is an outline of an attempted use of the integral test to determine whether the following series is convergent or divergent.

$$\sum_{k=1}^{\infty} ke^{-k^2}$$

(a) Determine which conditions of the integral test are satisfied by the function $f(x) = xe^{-x^2}$. **Select all that apply.** Mark the squares fully as directed below.

DO: ☐ ■
DON'T: ☑ ☒ ⧅ ▨ ◉ ■

WARNING: This is the only multi-select question on this exam. Questions 1–10 and the next part of this question are all single-select.

☐ (A) The function $f(x)$ is negative for $x \geq 1$.
☐ (B) The function $f(x)$ is positive for $x \geq 1$.
☐ (C) The function $f(x)$ is decreasing for $x \geq 1$.
☐ (D) The function $f(x)$ is increasing for $x \geq 1$.
☐ (E) The function $f(x)$ is continuous for $x \geq 1$.
☐ (F) The function $f(x)$ satisfies that $f(k) = a_k = ke^{-k^2}$.

(b) Select the correct conclusion below, and fill in the answer box to complete your choice. Use $+\infty$ or $-\infty$ as needed.

Note: To receive full credit, you need to both fill in a bubble to make a selection and fill in the answer box to complete the selected conclusion. Do not write anything in the other two answer boxes.

◯ (A) The series is convergent by the integral test, because $\int_1^{\infty} xe^{-x^2}\,dx =$ [ ].

◯ (B) The series is divergent by the integral test, because $\int_1^{\infty} xe^{-x^2}\,dx =$ [ ].

◯ (C) The integral test cannot be applied here, because condition(s) [ ] in part (a) is/are not satisfied. [*Enumerate appropriate letter(s) from* A–F.]

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12. For each of the following sequence or series, determine its limit or sum. Simplify numerical values as much as you can. If the limit or sum is infinite, write either $+\infty$ or $-\infty$ accordingly. If the limit or sum does not exist and does not approach one of the two infinities, write "DNE".

Note 1: If you write down ∞ without a sign, it will be interpreted as $+\infty$.
Note 2: If the limit or sum is $+\infty$ or $-\infty$, writing "DNE" will not receive credit.

(a) (3 points) The limit of the sequence $\left\{\frac{100^{2n}}{n!}\right\}_{n=1}^{\infty}$ is $\boxed{}$.

(b) (3 points) The sum of the series $\sum_{k=2}^{\infty} \frac{1}{3^k}$ is $\boxed{}$.

(c) (3 points) The limit of the sequence $\left\{\left(\cos(\pi n)\right)^2\right\}_{n=1}^{\infty}$ is $\boxed{}$.

(d) (3 points) The sum of the series $\sum_{k=1}^{\infty} \frac{1}{5k}$ is $\boxed{}$.

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