MA266 Practice Problems for Exam 1

1. If $y' + \left(1 + \frac{1}{t}\right) y = \frac{1}{t}$ and $y(1) = 0$, then $y(\ln 2) =$?

A. $\ln 2 - \ln(\ln 2)$ B. $\ln(\ln 2)$ C. $\ln(\ln 2) + \frac{1}{2\ln 2}$ D. $\frac{1}{\ln 2}\left(1 - \frac{e}{2}\right)$ E. $\frac{1}{\ln 2 - 1}$

2. What is the largest open interval for which a unique solution of the initial value problem

$$ty' + \frac{1}{t+1} y = \frac{t-2}{t-3}, \quad y(1) = 0$$

is guaranteed?

A. $0 < t < 1$ B. $0 < t < 2$ C. $0 < t < 3$ D. $-1 < t < 3$ E. $-1 < t < 1$

3. An explicit solution of $y' = y^2 - 1$ is?

A. $y = \frac{Ce^{2t}}{1 - Ce^{2t}}$ B. $y = \frac{1 + Ce^{2t}}{1 - Ce^{2t}}$ C. $y = \frac{1}{1 - Ce^{2t}}$ D. $y = \frac{1 + Ce^{2t}}{1 - e^{2t}}$ E. $\frac{y^3}{3} - y = C$

4. If $y' = y^3$ and $y(0) = 1$, then $y(-1) =$?

A. $5^{-\frac{1}{4}}$ B. $\frac{1}{\sqrt{3}}$ C. $\sqrt{3}$ D. 1 E. Does not exist

5. Let $y(x)$ be the solution to the initial value problem

$$xy' = 3y + 2x^4, \quad y(1) = 0.$$

Then, $y(2)$ is

A. 4 B. 8 C. 16 D. 20 E. 32

6. A tank initially contains 40 ounces of salt mixed in 100 gallons of water. A solution containing 4 oz of salt per gallon is then pumped into the tank at the rate of 5 gal/min. The stirred mixture flows out of the tank at the same rate. How much salt is in the tank after 20 minutes?

A. 20 B. 80 C. $40 + 20e$ D. $400 - 360e^{-1}$ E. $400 + 360e^2$

7. Find the general solution of a homogeneous equation using substitution $v = \frac{y}{x}$.

$$\frac{dy}{dx} = \frac{5x^2 + 3y^2}{2xy}$$

A. $y^2 + 5x^2 = Cx^3$ B. $3y^2 + 5x^2 = Cx^2$ C. $x^2 + 3y^2 = Cx$ D. $2y - 5x^2 = Cx^4$ E. $y^2 + 3x^2 = Cx^3$

8. Suppose that

$$\frac{dy}{dx} = (x + y)^2 - 1.$$

What is the implicit general solution to this differential equation? (Hint: use the substitution $v(x) = x + y$.)

A. $\frac{1}{x+y} + x = C$ B. $\frac{1}{x+y} - x = C$ C. $\frac{x}{y} + x = C$ D. $\frac{x}{y} - x = C$ E. $x(x + y) + 1 = C$

9. An implicit solution of

$$y^2 + 1 + (2xy + 1)\frac{dy}{dx} = 0$$

is?

A. $2(xy^2+y) = C$ B. $xy^2+y = C$ C. $xy^2+x+y = C$ D. $\frac{y^3}{3}+y+x^2y+x = C$ E. $y = xy^2+C$

10. Consider the autonomous differential equation

$$\frac{dy}{dt} = -\frac{1}{10}(y-1)(y-4)^2.$$

Classify the stability of each equilibrium solution.

A. $y = 1$ and $y = 4$ both unstable B. $y = 1$ unstable; $y = 4$ stable C. $y = 0$ and $y = 1$ stable; $y = 4$ unstable D. $y = 1$ stable; $y = 4$ semistable E. $y = 0$ stable; $y = 1$ and $y = 4$ unstable

11. Consider the following doomsday/extinction differential equation for a population $P(t)$ with the initial population $P(0) = 4$.

$$\frac{dP}{dt} = 3P(P-2)$$

At what time t does "Doomsday" occur (which means the population explodes)?

A. $\frac{\ln(2)}{6}$ B. $\frac{\ln(2)}{3}$ C. $\frac{\ln(4)}{3}$ D. $\frac{\ln(4)}{6}$ E. ∞

12. Use Euler's method with step size $h = 1$ to find the approximate value of $y(3)$, where $y(x)$ solves the initial value problem

$$y' = x + \frac{y}{2}, \quad y(0) = -8.$$

A. -17 B. -22.5 C. -23.5 D. -24.5 E. -27

13. If the Wronskian $W(f, g) = -3e^{4t}$ and $f(t) = 4e^{2t}$, then $g(t)$ could be

A. $-\frac{3}{4}te^{2t}$ B. $\frac{3}{4}te^{2t}$ C. $12e^{2t}$ D. $-\frac{3}{2}e^{2t}$ E. $-\frac{3}{4}te^{4t}$

14. The general solution of

$$y'' - 4y' + 4y = 0$$

is?

A. $y = C_1e^{2t} + C_2te^{2t}$ B. $y = C_1e^{2t} + C_2e^{2t}$ C. $y = C_1e^{2t} + C_2e^{-2t}$ D. $y = C_1e^{-2t} + C_2te^{-2t}$ E. $y = C_1t + C_2t^2$

15. The general solution of

$$y''' + 4y'' + 5y' = 0$$

is?

A. $y = C_1e^{-2t}\cos t + C_2e^{-2t}\sin t$ B. $y = C_1 + C_2e^{-2t}\cos t + C_3e^{-2t}\sin t$ C. $y = C_1 + C_2e^{t}\cos 2t + C_3e^{t}\sin 2t$ D. $y = C_1 + C_2\cos t + C_3\sin t$ E. $y = C_1 + C_2e^{2t}\cos t + C_3e^{2t}\sin t$

16. Let $y(x)$ be the solution to the reducible second-order differential equation

$$y'' + (y')^2 = 0, \;\; y(0) = 0, \; y'(0) = 1.$$

Find $y(2)$. (Use the substitution $p = y' > 0$.)

A. $\ln 3$ B. e^{-2} C. $\ln 5$ D. e^4 E. 4

17. An object weighting 8 pounds attached to a spring will stretch it 6 inches beyond its natural length. There is a damping force with a damping constant $c = 6$ lbs-sec/ft and there is no external force. If at $t = 0$ the object is pulled 2 feet below equilibrium and then released, the initial value problem describing the vertical displacement $x(t)$ becomes?

A. $\frac{1}{4}x'' + 6x' + 16x = 0, x(0) = 2, x'(0) = 0$ B. $8x'' + 6x' + 16x = 0, x(0) = -2, x'(0) = 0$

C. $8x'' + 6x' + 16x = 0, x(0) = 2, x'(0) = 0$ D. $\frac{1}{4}x'' + 6x' + 8x = 0, x(0) = 2, x'(0) = 0$ E. $256x'' + 6x' + 16x = 0, x(0) = 2, x'(0) = 0$

Answer Key: 1.D 2.C 3.B 4.B 5.C 6.D 7.A 8.A 9.C 10.D 11.A 12.C 13.A 14.A 15.B 16.A 17.A