MA266 Practice Problems for Final Exam

1. Find a particular solution of

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 2 \\ 3 \end{bmatrix}.$$

A. $\mathbf{x}_p = \begin{bmatrix} 2 \\ 3 \end{bmatrix}$ B. $\mathbf{x}_p = \begin{bmatrix} -2 \\ 3 \end{bmatrix}$ C. $\mathbf{x}_p = \begin{bmatrix} -3 \\ 2 \end{bmatrix}$ D. $\mathbf{x}_p = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$ E. $\mathbf{x}_p = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

2. Find the general solution of

$$\begin{bmatrix} x_1 \\ x_2 \end{bmatrix}' = \begin{bmatrix} 2 & 0 \\ 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} 6e^{-t} \\ 1 \end{bmatrix}.$$

A. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

B. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t}$

C. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} - \begin{bmatrix} 6e^{-t} \\ 1 \end{bmatrix}$

D. $c_1 \begin{bmatrix} 0 \\ 1 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 6 \\ 0 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

E. $c_1 \begin{bmatrix} 1 \\ 0 \end{bmatrix} e^t + c_2 \begin{bmatrix} 1 \\ 1 \end{bmatrix} e^{2t} + \begin{bmatrix} 2 \\ -1 \end{bmatrix} e^{-t} + \begin{bmatrix} 0 \\ 1 \end{bmatrix}$

3. $\mathcal{L}\{e^t(1 + \cos 2t)\} = ?$

A. $\dfrac{1}{s-1} + \dfrac{1}{(s-1)^2 + 4}$ B. $\dfrac{1}{s-1} \left(\dfrac{1}{s} + \dfrac{s-1}{(s-1)^2 + 4} \right)$ C. $\dfrac{1}{s-1} \dfrac{s-1}{s^2 - 2s + 5}$ D. $\dfrac{1}{s} + \dfrac{s}{(s-1)^2 + 4}$
E. $\dfrac{1}{s-1} + \dfrac{s-1}{s^2 - 2s + 5}$

4. Find the Laplace transform of

$$f(t) = \begin{cases} t, & 0 \le t < 1 \\ 0, & 1 \le t < \infty \end{cases}.$$

A. $e^{-s} \left(\dfrac{1}{s} + \dfrac{1}{s-2} \right)$ B. $\dfrac{1}{s^2} - e^{-s} \dfrac{1}{s^2}$ C. $\dfrac{1}{s^2} - e^{-s} \left(\dfrac{1}{s} + \dfrac{1}{s^2} \right)$ D. $\dfrac{1}{s^2} + 2e^{-s} \left(\dfrac{1}{s} + \dfrac{1}{s^2} \right)$
E. $e^{-s} \left(\dfrac{1}{s} + \dfrac{1}{s^2} \right)$

5. Solve

$$y'' + 3y' + 2y = 4u_1(t)$$
$$y(0) = 0, \quad y'(0) = 1.$$

A. $u_1(t) \left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)} \right)$

B. $u_1(t) \left(2 - 4e^{-(t-1)} + 2e^{-2(t-1)} \right) + e^{-t} - e^{-2t}$

C. $u_0(t)\left(2-4e^{-(t-1)}+2e^{-2(t-1)}\right)+e^{-t}-e^{-2t}$

D. $\left(2-4e^{-(t-1)}+2e^{-2(t-1)}\right)+e^{-t}-e^{-2t}$

E. $e^{-t}-e^{-2t}$

6. Find the solution of the initial value problem

$$\begin{aligned} &y''+y=\delta(t-\pi)\\ &y(0)=0, \quad y'(0)=1. \end{aligned}$$

A. $y = \sin t + u_0(t)\sin(t-\pi)$ B. $y = \sin t + u_\pi(t)\sin(\pi t)$ C. $y = u_\pi(t)(\sin t + \sin(t-\pi))$
D. $y = u_\pi(t)\sin t$ E. $y = \sin t + u_\pi(t)\sin(t-\pi)$

7. The inverse Laplace transform of

$$F(s)=\frac{se^{-s}}{s^2+2s+5}$$

is?

A. $u_1(t)\left(e^{t-1}\cos 2(t-1)-\frac{1}{2}e^{t-1}\sin 2(t-1)\right)$

B. $u_1(t)\left(e^{-t}\cos 2t\right)-\frac{1}{2}e^{-t}\sin 2t$

C. $u_1(t)\left(e^{-t+1}\cos 2(t-1)-\frac{1}{2}e^{-t+1}\sin 2(t-1)\right)$

D. $u_1(t)\left(e^{-t}\cos 2(t-1)-\frac{1}{2}e^{-t}\sin 2(t-1)\right)$

E. $e^{-t+1}\cos 2(t-1)-\frac{1}{2}e^{-t+1}\sin 2(t-1)$

8. $\mathcal{L}\left\{\int_0^t \sin 2(t-\tau)\cos(3\tau)d\tau\right\}=?$

A. $\dfrac{1}{s^2+4}+\dfrac{s}{s^2+9}$ B. $\dfrac{2s}{(s^2+4)(s^2+9)}$ C. $\dfrac{2}{s^2+4}+\dfrac{s}{s^2+9}$ D. $\dfrac{2}{(s^2+4)(s^2+9)}$ E. $\dfrac{s}{(s^2+4)(s^2+9)}$

Answer Key: 1.D 2.A 3.E 4.C 5.B 6.E 7.C 8.B