MA266 Practice Problems for Exam 2

1. A particular solution, y_p, of

$$y'' - 4y' + 3y = 2t + e^t$$

is?

A. $-\frac{1}{2}te^t + \frac{1}{3}t + \frac{1}{2}$ B. $-\frac{1}{2}te^t + \frac{1}{2}t + \frac{1}{2}$ C. $-\frac{1}{2}e^t + \frac{1}{3}t + \frac{1}{2}$ D. $t^2 + e^t$ E. $-\frac{1}{2}te^t + \frac{2}{3}t + \frac{8}{9}$

2. Determine the appropriate form for a particular solution $y_p(x)$ to the third-order differential equation

$$y^{(3)} + y'' - y' - y = \cos x + xe^{-x}.$$

A. $A\cos x + B\sin x + x^2(Cx+D)e^{-x}$ B. $A\cos x + x(Bx+C)e^{-x}$ C. $x^2(A\cos x + B\sin x) + (Cx+D)e^{-x}$
D. $A\cos x + Bxe^{-x}$ E. $A\cos x + B\sin x + (Cx+D)e^{-x}$

3. If $y'' + 5y' + 6y = 24e^t$, $y(0) = 0$, $y'(0) = 0$, then $y(1) =$?

A. $e - e^{-2} + 6e^{-3}$ B. $2e - 8e^{-2} + 6e^{-3}$ C. $e - 8e^{-2} + 6e^{-3}$ D. $e + 8e^{-2} + e^{-3}$ E. 0

4. The differential equation

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 0$$

has solutions $y_1(t) = t$ and $y_2(t) = t^2$. If

$$y'' - \frac{2}{t}y' + \frac{2}{t^2}y = 2; \quad y(1) = 0,\ y'(1) = 0$$

then $y(2) =$?

A. $8\ln 2 - 4$ B. 0 C. -6 D. $8\ln 2 + 4$ E. $8\ln 2$

5. A spring-mass system is governed by the initial value problem

$$x'' + 4x' + 4x = 4\cos\omega t$$
$$x(0) = 9, \quad x'(0) = -2.$$

For what value(s) of ω will resonance occur?

A. 0 B. 2 C. 4 D. no value of ω E. $2 < \omega < \inf$

6. Rewrite the second order equation

$$2u'' + 3u' + ku = \cos 2t$$

as a system of first order equations.

A. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3x - ky + \cos 2t) \end{cases}$ B. $\begin{cases} x' = x \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$ C. $\begin{cases} x' = y \\ y' = \frac{1}{2}(-3y - kx + \cos 2t) \end{cases}$

D. $\begin{cases} x' = y \\ y' = 2y + kx + \cos 2t \end{cases}$ E. $\begin{cases} x' = 2y + kx + \cos 2t \\ y' = x \end{cases}$

7. The solution of

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ 4 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 3 \\ 2 \end{bmatrix}$$

is?

A. $2e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$ B. $2e^{3t} \begin{bmatrix} 1 \\ 0 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ C. $e^{3t} \begin{bmatrix} 2 \\ 1 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ D. $3e^{3t} \begin{bmatrix} 1 \\ -2 \end{bmatrix} - e^{-t} \begin{bmatrix} 1 \\ -2 \end{bmatrix}$

E. $3e^{3t} \begin{bmatrix} 1 \\ 2 \end{bmatrix} + e^{-t} \begin{bmatrix} 0 \\ -4 \end{bmatrix}$

8. Solve

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}.$$

A. $\mathbf{x}(t) = 2e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ B. $\mathbf{x}(t) = 2e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} + e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ C. $\mathbf{x}(t) = 2e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ -\sin t \end{bmatrix}$

D. $\mathbf{x}(t) = e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$ E. $\mathbf{x}(t) = e^t \begin{bmatrix} -\sin t \\ \cos t \end{bmatrix} - e^t \begin{bmatrix} \cos t \\ \sin t \end{bmatrix}$

9. Solve the initial value problem

$$\mathbf{x}' = A\mathbf{x}, \quad \mathbf{x}(0) = \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \quad \text{where } A = \begin{bmatrix} 1 & 1 \\ 0 & 1 \end{bmatrix}.$$

A. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$ B. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$ C. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} + te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$ D. $e^t \begin{bmatrix} 0 \\ 1 \end{bmatrix} + 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

E. $e^t \begin{bmatrix} 1 \\ 1 \end{bmatrix} - 2te^t \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$

10. What values of the parameter α in the system below make the origin a saddle point in the phase plane:

$$\mathbf{x}' = \begin{bmatrix} 1 & 1 \\ \alpha & 2 \end{bmatrix} \mathbf{x}$$

A. $\alpha > 2$ B. $\alpha > -\frac{1}{4}$ C. $\alpha < -\frac{1}{4}$ D. $2 > \alpha > -\frac{1}{4}$ E. $\alpha < -2$

Answer Key: 1.E 2.A 3.B 4.A 5.D 6.C 7.A 8.C 9.C 10.A