MATH 265 – HANDWRITTEN HOMEWORK 26

1. Express the quotient $z = \dfrac{1+3\mathrm{i}}{6+8\mathrm{i}}$ as $z = re^{\mathrm{i}\theta}$.

2. Express $z = 10e^{\mathrm{i}\frac{\pi}{6}}$ as $z = a + b\mathrm{i}$.

3. Find all values of r such that the complex number $re^{\mathrm{i}\frac{\pi}{4}} = a + b\mathrm{i}$, where both a and b are integers.

4. Find all read and complex roots of the equation $z^{10} = 9^{10}$.

5. Find all real and complex solutions to the equation $x^4 - 2x^2 + 1 = 0$.

6. Find all real and complex eigenvalues of the matrix

$$A = \begin{bmatrix} 4 & 0 & 0 \\ 0 & 1 & -1 \\ 0 & 5 & -3 \end{bmatrix}.$$

7. Show that if $p(x)$ is a polynomial with real coefficients and z is a solution of $p(x) = 0$, then $\bar{z}$ is also a solution of $p(x) = 0$, that is $p(\bar{z}) = 0$.

8. One can identify complex numbers and vectors on the plane through $\mathbb{R}^2$ by $a + b\mathrm{i} \equiv \begin{bmatrix} a \\ b \end{bmatrix}$.

   Using this identification, find the matrix $B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}$ such that

$$e^{\mathrm{i}\phi}(a + b\mathrm{i}) \equiv B \begin{bmatrix} a \\ b \end{bmatrix}.$$

   Use this to explain geometrically the action of the matrix B on the vector $\begin{bmatrix} a \\ b \end{bmatrix}$.