Functions Of A Complex Variable I
Instructor: Professor Gregery Buzzard
Course Number: MA 53000
Credits: Three
Time: 8:30–9:20 AM MWF

Catalog Description

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics. More mathematically rigorous than MA 52500.

Elements Of Stochastic Processes
Instructor: Professor Christopher Janjigian
Course Number: MA 53200
Credits: Three
Time: 9:30–10:20 AM MWF

Catalog Description

A basic course in stochastic models, including discrete and continuous time Markov chains and Brownian motion, as well as an introduction to topics such as Gaussian processes, queues, epidemic models, branching processes, renewal processes, replacement, and reliability problems.

Probability Theory I
Instructor: Professor Samy Tindel
Course Number: MA 53800
Credits: Three
Time: 4:30–5:45 PM TTh

Description

This is a basic course on stochastic models. We will first review different modes of convergence for random variables. Then we will explore some widely used classes of stochastic processes: martingales, renewal processes, stationary processes and queues. This will be done following:

Textbook: Grimmett, Stirzacker: Probability and Random Processes. Oxford University Press, 2020.

Ordinary Differential Equations And Dynamical Systems
Instructor: Professor Nung Kwan Yip
Course Number: MA 54300
Credits: Three
Time: 12:00–1:15 PM TTh

Catalog Description

This course focuses on the theory of ordinary differential equations and methods of proof for developing this theory. Topics include basic results for linear systems, the local theory for nonlinear systems (existence and uniqueness, dependence on parameters, flows and linearization, stable manifold theorem) and the global theory for nonlinear systems (global existence, limit sets and periodic orbits, Poincare maps). Permission of instructor required.

Real Analysis And Measure Theory
Instructor: Professor Antônio Sá Barreto
Course Number: MA 54400
Credits: Three
Time: 11:30 AM–12:20 PM MWF

Description

This is a qualifying exam course and I will follow the qualifying exam syllabus (which can be found on the grad student handbook) very closely.

Part A: A Short review of topics in undergraduate level analysis and a bit more:

1.1 The topology of $\mathbb{R}^n$ Convergence of sequences, dense sets. Metric spaces.

2.2 Continuity, uniform continuity and semi-continuity

3.3 Sequences of functions, pointwise and uniform convergence

4.4 The Riemann integral, sets of content zero and sets of measure measure zero. Cantor sets.

5.5 When is $\lim_{n\to\infty} \int_a^b f_n \, dx = \int_a^b \lim_{n\to\infty} f_n \, dx$ for the Riemann integral?

Part B: Abstract Measures

B.1 Abstract measure spaces and integral properties of measurable functions

B.2 Notions of convergences. Convergence in measure, almost everywhere convergence, etc.

B.3 Convergence theorems, Fatou's Lemma

B.4 L^p- spaces, Hölder and Cauchy-Schwatz inequalities. Banach and Hilbert spaces.

B.5 The Fubini-Tonelli Theorem

Part C: The Lebesgue Measure in $\mathbb{R}^n$

C .1 The construction of the Lebesgue measure and integral in $\mathbb{R}^n$

C .2 When is $\lim_{n\to\infty} \int f_n \, dx = \int \lim_{n\to\infty} f_n \, dx$ for the Lebesgue integral?

C .3 The Fubini-Tonelli Theorem revisited

C .4 Applications: Convolutions and approximations of the identity;

C .5 The Fourier transform and its inverse. The Fourier transform of L^2 functions

Part D: Differentiation of functions

D.1 The Vitali Covering Theorem

D.2 Functions of bounded variation, differentiation of monotone functions, absolute continuity, the Helly Selection Theorem

D.3 The Lebesgue Differentiation Theorem

References:

E.1 I will type my own course notes (for which I claim no originality) which will be posted in Brightspace.

E.2 I will also post my handwritten lecture notes in Brightspace.

E.3 There will be no textbook. References: A. Torchinsky, Real Variables and W. Rudin, Real and Complex Analysis

Grade:

1. One set of homework problems per week. Average of the homework scores= 150 points. No scores will be dropped. Students are highly encouraged to work together on homework assignments.

2. Two evening midterm exams (two hours long) –100 points each. Students will not be allowed to collaborate with each other or consult notes or books during the exams

3. Final exam– 150 points

4. Total: 500 points.

5. Grade curve (which can be adjusted at the end of the semester. I may lower the cut-offs, but I will not raise them):
   A range: 425-449, A minus – 450- 474, A – 475-500, A plus
   B range: 350- 374, B minus– 375- 399, B – 400- 424, B plus
   C range: 275– 299, C minus – 300 - 259, C – 325- 349, C plus
   D range: 250- 274.
   F range: < 250

Introduction To Functional Analysis
Instructor: Professor Kiril Datchev
Course Number: MA 54600
Credits: Three
Time: 12:30–1:20 PM MWF

Catalog Description

Fundamentals of functional analysis. Banach spaces, Hahn-Banach theorem. Principle of uniform boundedness. Closed graph and open mapping theorems. Applications. Hilbert spaces. Orthonormal sets. Spectral theorem for Hermitian operators and compact operators.

Introduction To Abstract Algebra
Instructor: Professor Saugata Basu
Course Number: MA 55300
Credits: Three
Time: 10:30–11:20 AM MWF

Catalog Description

Group theory: Sylow theorems, Jordan-Holder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

Linear Algebra I
Instructor: Professor Bernd Ulrich
Course Number: MA 55400
Credits: Three
Time: 1:30–2:20 PM MWF

Catalog Description

Review of basics: vector spaces, dimension, linear maps, matrices determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

Abstract Algebra II
Instructor: Professor Tong Liu
Course Number: MA 55800
Credits: Three
Time: 10:30–11:20 AM MWF

Catalog Description

This course is a continuation of MATH 557. The course will cover topics in (homological) dimension theory, regular sequences, Tor and Ext, Koszul complex, local cohomology, Cohen-Macaulay rings, and Gorenstein rings. The course should be accessible to anyone who has done MATH 557 or has a working knowledge of the text in Atiyah–McDonald. No particular book will be followed, but we will mainly use the following references: Commutative ring theory by Matsumura, An introduction to homological algebra by Weibel, `https://dept.math.lsa.umich.edu/~hochster/cmrvw.pdf`

The Course grade is determined by homework and attendance.

Introduction In Algebraic Topology
Instructor: Professor David Ben McReynolds
Course Number: MA 57200
Credits: Three
Time: 12:00–1:15 PM TTh

Catalog Description

Singular homology theory; Eilenberg-Steenrod axioms; simplicial and cell complexes; elementary homotopy theory; Lefschetz fixed point theorem.

Algebraic Number Theory
Instructor: Professor Baiying Liu
Course Number: MA 58400
Credits: Three
Time: 9:30–10:20 AM MWF

Catalog Description

Dedekind domains, norm, discriminant, different, finiteness of class number, Dirichlet unit theorem, quadratic and cyclotomic extensions, quadratic reciprocity, decomposition and inertia groups, completions and local fields.

Introduction to Stochastic Calculus
Instructor: Professor Christopher Janjigian
Course Number: MA 59500BM
Credits: Three
Time: 12:30–1:20 PM MWF

Description

An introductory course on Brownian motion and Brownian stochastic calculus. Topics will include basic properties of Gaussian random variables, basic path properties of Brownian motion, basic properties of martingales, basic properties of Ito stochastic integrals and stochastic differential equations, the (strong) Markov property, connections to partial differential equations including the Kolmogorov and Fokker-Planck equations, and the Cameron-Martin and Girsanov theorems. Further topics may include some applications to mathematical finance, stochastic control, or stochastic filtering.

Textbook:
A First Course in Stochastic Calculus by Arguin. ISBN: 978-1-4704-6488-2

Prerequisites: (Required)
Basic probability at the level of MATH/STAT 416 or 519
Ordinary differential equations at the level of MATH 266

(Recommended):
Some exposure to stochastic processes at the level of 432/532.

Filtering Complex Fluid Systems
Instructor: Professor Di Qi
Course Number: MA 59500FC
Credits: Three
Time: 11:30 AM–12:00 PM MWF

Description

Filtering (also known as data assimilation) offers an innovative tool for finding the optimal probability distribution (the posterior) of the unobserved states and model parameters by combining dynamical model forecast (the prior) with certain partially observed data with noises. This advanced topic course will discuss filtering noisy turbulent signals for complex dynamical systems through an applied mathematics perspective involving the blending of rigorous mathematical theories, qualitative and quantitative modeling, and novel numerical procedures. The course will begin with an elementary introduction to these topics including classical analysis for SDEs and PDEs and their computational approximations, followed by data assimilation methods including Kalman filtering, ensemble Kalman filters, and instructive stochastic qualitative models from turbulence theory and concrete models such as from climate atmosphere ocean science. Recent development in new mathematical theories and algorithms for fully nonlinear dynamical systems will also be discussed.

Infinite Dimensional Lie Algebras and Applications
Instructor: Professor Oleksandr Tsymbaliuk
Course Number: MA 59500IDL
Credits: Three
Time: 12:00–1:15 PM TTh

Description

This course is a detailed introduction into the structure and representation theory of some of the most important infinite dimensional Lie algebras: Heisenberg algebras, Kac-Moody algebras, and Virasoro algebra. The course is expected to fit a wide range of students: graduate and strong undergraduate mathematics students, as well as graduate physics students.

Tentative list of topics:

- Heisenberg algebra, Virasoro algebra, and affine $\widehat{\mathfrak{g}}$ as universal central extensions
- Representations of the Heisenberg algebra, the Virasoro algebra, and affine $\widehat{\mathfrak{sl}}_n$ via Lie algebras $\mathfrak{gl}_\infty$, $\mathfrak{a}_\infty$, and application to integrable systems
- Boson-fermion correspondence: vertex operator construction and Schur polynomials
- Feigin-Fuchs-Kac determinant formula for Virasoro and the region of unitarity
- The Sugawara construction and the Goddard-Kent-Olive construction
- Structure and representation theory of Kac-Moody algebras
- The Weyl-Kac character formula and the Kac-Macdonald identities
- Shapovalov-Jantzen-Kac-Kazhdan determinant formula for Kac-Moody algebras

Prerequisites: Basic notions from algebra (especially linear algebra). Familiarity with basic results on finite-dimensional Lie algebras is welcomed but not mandatory.

Introduction to Number Theory

Instructor: Professor Trevor D. Wooley
Course Number: MA 59500INT
Credits: Three
Time: 4:30–5:45 PM TTh

Description

Prerequisites: This course is intended for third- or fourth-year undergraduate students or beginning graduate students who have taken and obtained a grade of B- or better in MA 35301 (Linear Algebra II). Students should have basic competence in mathematical proof.

Number Theory studies the properties of integers, and includes the theory of prime numbers, the arithmetic structures that underlie cryptosystems such as RSA, Diophantine equations (polynomial equations to be solved in integers, including the topic of Fermat's Last Theorem), and rational approximations that distinguish algebraic and transcendental numbers. Although a topic studied for more than two millenia, it is the subject of intense active current research, and connects with many other areas of Mathematics.

This course serves as an introductory exploration of Number Theory, without an abstract algebra prerequisite, so that final year students without a pure mathematics background will find this accessible. Connections with abstract algebra will, however, be noted for interested students, and the material should provide reinforcement and preparation for abstract algebra for those with ambitions in this direction.

Content: We begin with a reasonably brisk discussion of the basic notions: the Euclidean algorithm, primes and unique factorisation, congruences, Chinese Remainder Theorem (Public Key Cryptosystems), primitive roots, quadratic reciprocity, arithmetic and multiplicative functions. The second part of the course is devoted to topics: binary quadratic forms, Diophantine approximation and transcendence, continued fractions, Pell's equation and other Diophantine equations, and quadratic fields (subject to time constraints).

Companion Text: An Introduction to the Theory of Numbers (Niven, Zuckerman and Montgomery, 5th edition, Wiley, 1991.)

The course will be based on the instructor's comprehensive web-page hosted LaTeXed notes.

Assessment: Course credit will be based on weekly homeworks – the top 10 scores are totalled; two mid-terms and final exam.

Topics in Model Theory
Instructor: Professor Margaret Thomas
Course Number: MA 59500MT
Credits: Three
Time: 10:30–11:20 AM MWF

Description

This course serves as an introduction to model theory (a branch of mathematical logic), which is the study of mathematical structures in terms of their logical properties. Some of the central questions that one can investigate from this perspective include the following. Given a mathematical structure, such as the real or complex field (often enriched with additional operations), one can analyse its class of definable sets (that is, the sets that can be defined in that structure using particular kinds of logical expressions). In particular, one can study the algebraic/geometric/combinatorial nature of these definable sets in terms of the logical complexity of their defining expressions, or seek to understand whether or not certain interesting sets/functions can be realised as definable sets therein. Alternatively, one can study the class of all structures in which certain logical expressions are true, considering, for example, how many such structures there can be of any given size, up to isomorphism. By bringing a foundational perspective to core mathematical ideas, model theory can be (and has been!) applied to many other areas of mathematics (including algebra, combinatorics, algebraic geometry, number theory, operator theory, dynamical systems, ...) and beyond.

The goal of this course is to cover a variety of central concepts in model theory, in particular motivated by various areas of application. Such concepts could include elementary extensions, ultraproducts, complete theories, categoricity, model completeness, quantifier elimination, elimination of imaginaries, types, saturated and homogeneous models, indiscernibles and stability. The aim will also be to discuss some of the key examples of model-theoretic structures and their properties, which could include algebraically closed fields (and more generally strongly minimal and stable theories) and real closed fields (and more generally o-minimal and NIP theories), with a view to modern applications of model theory (including applications of o-minimal structures to diophantine geometry, Hodge theory, and dynamical systems; and the role of NIP and related tame structures in combinatorics).

Prerequisites: Key background from a first course in logic (such as MA 58500) will be assumed (including, but not necessarily limited to, first-order formulae, structures and definable sets; the completeness theorem and compactness theorem of first-order logic; and some set theory background such as infinite cardinalities). An awareness of certain concepts from abstract algebra will also be helpful as these will be used to illustrate certain key ideas.

Computational Optimal Transport and Deep Generative Models
Instructor: Professors Rongjie Lai
Course Number: MA 59500OT
Credits: Three
Time: 10:30–11:20 AM MWF

Description

Optimal Transport has gained significant attention in recent years across a range of applications, particularly in machine learning and deep learning. This course explores the computational aspects of optimal transport and its variants. Topics will include the theoretical foundation of optimal transport and the Wasserstein distance, along with numerical algorithms such as linear programming, duality formulations, and Sinkhorn's algorithm. A key focus will be on the dynamic formulation of optimal transport and variational PDE-based algorithms. Additionally, the course will delve into connections between optimal transport and various deep generative models, including Generative Adversarial Networks (GANs), normalizing flows, and diffusion models. By the end of the course, students will have a deep understanding of both the theory and practical algorithms for optimal transport, as well as how these methods integrate with modern deep learning models.

Radon Transforms
Instructor: Professor Plamen Stefanov
Course Number: MA 59500RT
Credits: Three
Time: 3:00–4:15 PM TTh

Description

The Radon transform R maps a function f to its integrals along all (hyper-)planes.

The associated X-ray transform X integrates f along all lines. A fundamental problem is the inversion of those transforms in various situations: with full data (there are explicit formulas then), with incomplete, respectively discrete data, in presence of noise, etc. Studied first by Radon, and rediscovered by A. Cormack and G. Hounsfield (the 1979 Nobel prize in Physiology and Medicine), the inversion of the X-ray transform is the mathematical model of CT (Computed Tomography) scan, also known as CAT scan. More general transforms, like the X-ray transform over geodesics of a certain metric appear in various applications, for example in seismology, and is of its own interest in geometry.

We will start and stay mostly with the Euclidean case. The first part of the course will study the mapping properties of R and X, extension to distributions (which I will briefly introduce for those not familiar with them), inversion formulas, stability estimates, range conditions, support theorems, recovery in a region of interest with incomplete data. We will study the X-ray transforms of tensor fields, as well, and explain the motivation. If time permits, I will introduce the light-ray transform: integrals of functions $f(t,x)$ over light-rays in the Minkowski metric, and discuss its invertibility.

The second part of the course will concentrate on the weighted X-ray transform and microlocal considerations. I will introduce some microlocal concepts briefly and explain what they predict about recovery of singularities (e.g., edges) with incomplete data, in particular. Numerical examples will be presented.

The course should be accessible to students having good analysis background, including some familiarity with functional analysis (Hilbert spaces, linear operators but no deep knowledge is required), and the Fourier transform. I will follow a book by me and G. Uhlmann which I will make available online (an older version is on my website even now). This book is still not published. Relevant books for the first part of the course are also the classical book by Helgason "Radon Transform", available for free on his webpage, and Natterer's book "The Mathematics of Computerized Tomography."

Finite Element Methods for Partial Differential Equations

Instructor: Professor Zhiqiang Cai

Course Number: MA/CS 61500

Credits: Three

Time: 12:00–1:15 PM TTh

Description

The finite element method is the most widely used numerical technique in computational science and engineering. This course covers the basic mathematical theory of the finite element method for partial differential equations (PDEs) including variational formulations of PDEs and construction of continuous finite element spaces. Adaptive finite element method as well as fast iterative solvers such as multigrid and domain decomposition for algebraic systems resulting from discretization will also be presented. When time permits, neural network as a new class of approximating functions will also be covered.

Prerequisite: MA/CS 514 or equivalent or consent of instructor.

References

[1] S. Brenner and R. Scott, The Mathematical Theory of Finite Element Methods, Springer-Verlag, New York, 2002.

[2] D. Braess, Finite Elements: Theory, Fast Solvers, and Applications in Solid Mechanics, Cambridge University Press, New York, 1997.

[3] C. Johnson, Numerical Solution of Partial differential Equations by the Finite Element Method, Cambridge University Press, Cambridge, 1987.

Algebraic Geometry II
Instructor: Professor Takumi Murayama
Course Number: MA 665
Credits: Three
Time: 10:30–11:45AM TTh

Description

This course is the second course in a two semester introductory sequence in algebraic geometry. Algebraic geometry is the geometric study of solutions to systems of polynomial equations. Algebraic geometry has interactions with many other fields of mathematics, including commutative algebra, algebraic topology, number theory, several complex variables, and complex geometry.

This second course will mainly focus on the theory of schemes, including the necessary background on sheaves and their cohomology. Planned topics (subject to change) include the following: Sheaves of Abelian groups. Locally ringed spaces and sheaves of modules. Schemes, properties of schemes. Separated, proper, and projective morphisms of schemes. Cartier and Weil divisors. Sheaves of differentials. Derived functors and sheaf cohomology. Čech cohomology, cohomology of projective space. Ext groups and sheaves. Serre duality. Higher direct images. Flat morphisms. Smooth morphisms. Formal schemes, the theorem on formal functions. The semicontinuity theorem. Applications to curves and surfaces.

Prerequisites: MA 55300, 55400, 55700, 56200, 57100, 57200, and 59500AG.

Text: Course notes will be provided. The notes will largely draw from *Algebraic geometry* by Robin Hartshorne (available at `https://doi.org/10.1007/978-1-4757-3849-0` via the Purdue library).

Optional texts: All texts listed below have free access options for Purdue students.

- *Éléments de géométrie algébrique* by Alexander Grothendieck and Jean Dieudonné (available at `http://www.numdam.org`).
- *Eléments de géométrie algébrique I* (second edition) by Alexander Grothendieck and Jean Dieudonné (available for short term loan at `https://n2t.net/ark:/13960/t42s6kw4b`).

On Mapping Class Group
Instructor: Professors Sam Nariman, Lvzhou Chen
Course Number: MA 69700
Credits: Three
Time: 10:30–11:45 AM TTh

Description

This course will be about the mapping class groups of surfaces (i.e. the groups of homeomorphisms of surfaces up to isotopy), their interesting subgroups (e.g. Torelli, surface braid groups, etc), their dynamical properties like Nielsen-Thurston classification of elements of the mapping class group, their cohomological properties and their relation to the moduli space of Riemann surfaces. We will start with basic geometric group theory properties of the mapping class group and their finite presentations. To do so, we will discuss various complexes of curves on surfaces on which the mapping class group acts to study the properties of this group. We will prove the Dehn-Nielsen-Baer theorem that relates the mapping class group to the automorphisms of the fundamental group. After discussing the geometry of surfaces and a little bit of Teichmuller's theory, we will discuss the Nielsen-Thurston classification of elements of the mapping class group.

Depending on the time and interest, after these classical topics, we will venture into more recent topics on mapping class groups of surfaces. Such topics could include the proof of Mumford's conjecture on the cohomology of the moduli space, big mapping class groups, fine curve complexes, homeomorphisms of surfaces, etc.

We assume some basic knowledge of algebraic topology, differential topology, manifolds, and group actions. Some familiarity with hyperbolic geometry can be helpful but is not required.