

Introduction to Number Theory

Instructor: Professor Alisa Sedunova

Course Number: MA 49500NT

Credits: Three

Time: 12:00–1:15 PM TTh

Prerequisites:

This course is intended for third- or fourth-year undergraduate students or beginning graduate students who have taken and obtained a grade of B– or better in MA 35301 (Linear Algebra II). Students should have basic competence in mathematical proof.

Description

Number Theory studies the properties of integers, and includes the theory of prime numbers, the arithmetic structures that underlie cryptosystems such as RSA, Diophantine equations (polynomial equations to be solved in integers, including the topic of Fermat’s Last Theorem), and rational approximations that distinguish algebraic and transcendental numbers. Although a topic studied for more than two millennia, it is the subject of intense active current research, and connects with many other areas of Mathematics.

This course serves as an introductory exploration of Number Theory, without an abstract algebra prerequisite, so that final-year students without a pure mathematics background will find this accessible. Connections with abstract algebra will, however, be noted for interested students, and the material should provide reinforcement and preparation for abstract algebra for those with ambitions in this direction.

Content

The course will broadly follow the structure and spirit of *An Introduction to the Theory of Numbers* by Hardy and Wright, with selections adapted to the level and objectives of the class.

We begin with the fundamental properties of prime numbers, the Euclidean algorithm, unique factorization, and the theory of congruences, including the Chinese Remainder Theorem. We then proceed to the multiplicative structure of the integers modulo m , primitive roots, Legendre and Jacobi sym-

bols, quadratic reciprocity, illustrative examples of quadratic congruences, and arithmetic and multiplicative functions.

Subsequently, the Prime Number Theorem will be stated without proof, with emphasis placed on its consequences and general significance, while establishing several weaker yet non-trivial estimates for the prime-counting function(s).

The latter part of the course is devoted to binary quadratic forms and their role in the representation of integers, together with selected topics in Diophantine approximation and transcendence, continued fractions, Pell's equation, aselected remarks related to Fermat's Last Theorem (subject to time constraints).

Companion Text(s)

The following is a list of sources used by the instructor in preparing this course. Students are welcome to consult any of these references in addition to the lecture notes, although it is neither necessary nor expected that they study all of them in detail.

(1) *Introduction to Analytic Number Theory*, by Tom M. Apostol, Springer, 1976.

(2) *An Introduction to the Theory of Numbers*, by G. H. Hardy and E. M. Wright, 6th edition, Oxford University Press, 2008.

(3) *The Distribution of Prime Numbers*, by Dimitris Koukoulopoulos, Graduate Studies in Mathematics, Vol. 203, American Mathematical Society, 2019.

(4) *An Introduction to the Theory of Numbers*, by Ivan Niven, Herbert S. Zuckerman, and Hugh L. Montgomery, 5th edition, Wiley, 1991.

(5) *Introduction to Analytic and Probabilistic Number Theory*, by Gérald Tenenbaum, 3rd edition, American Mathematical Society, 2015.

The main textbook would be *An Introduction to the Theory of Numbers*, by G. H. Hardy and E. M. Wright, 6th edition, Oxford University Press,

2008. If for some reason this does not work, one can use *An Introduction to the Theory of Numbers*, by Ivan Niven, Herbert S. Zuckerman, and Hugh L. Montgomery, 5th edition, Wiley, 1991 as the textbook as well (there is a large overlap anyways).

The course will be based on the instructor's notes distributed via brightspace, the HWs are to be submitted there as well.

Assessment

Course credit will be based on bi-weekly homeworks — the top 5 scores are totalled; two in class mid-terms and final exam.