

Spring 2027 Graduate Mathematics Course Descriptions

Purdue University — Department of Mathematics

MA 53000 Functions Of A Complex Variable I

Instructor: Professor Steven Bell

Course Number: MA 53000

Credits: Three

Meeting Time: 2:30–3:20 PM Monday / Wednesday / Friday

Catalog Description

Complex numbers and complex-valued functions of one complex variable; differentiation and contour integration; Cauchy's theorem; Taylor and Laurent series; residues; conformal mapping; special topics. More mathematically rigorous than MA 52500.

MA 53200 Elements of Stochastic Processes

Instructor: Professor Ajay Chandra

Course Number: MA 53200

Credits: Three

Meeting Time: 9:00–10:15 AM Tuesday / Thursday

Catalog Description

(STAT 53200) A basic course in stochastic models, including discrete and continuous time Markov chains and Brownian motion, as well as an introduction to topics such as Gaussian

processes, queues, epidemic models, branching processes, renewal processes, replacement, and reliability problems.

MA 53800 Probability Theory I

Instructor: Professor Ajay Chandra

Course Number: MA 53800

Credits: Three

Meeting Time: 10:30–11:45 AM Tuesday / Thursday

Catalog Description

(STAT 53800) Mathematically rigorous, measure-theoretic introduction to probability spaces, random variables, independence, weak and strong laws of large numbers, conditional expectations, and martingales.

MA 54300 Ordinary Differential Equations and Dynamical Systems

Instructor: Professor Aaron N Yip

Course Number: MA 54300

Credits: Three

Meeting Time: 10:30–11:45 AM Tuesday / Thursday

Catalog Description

This course focuses on the theory of ordinary differential equations and methods of proof for developing this theory. Topics include basic results for linear systems, the local theory for nonlinear systems (existence and uniqueness, dependence on parameters, flows and linearization, stable manifold theorem) and the global theory for nonlinear systems (global existence, limit sets and periodic orbits, Poincare maps). Permission of instructor required.

MA 54400 Real Analysis And Measure Theory

Instructor: Professor Antônio Sâ Barreto

Course Number: MA 54400

Credits: Three

Meeting Time: 12:30–1:20 PM Monday / Wednesday / Friday

Description

This is a qualifying exam course and I will follow the qualifying exam syllabus (which can be found on the grad student handbook) very closely.

Part A. A Short review of topics in undergraduate level analysis and a bit more:

- A.1. The topology of $\mathbb{R}^n$. Convergence of sequences, dense sets. Metric spaces.
- A.2. Continuity, uniform continuity and semi-continuity
- A.3. Sequences of functions, pointwise and uniform convergence
- A.4. The Riemann integral, sets of content zero and sets of measure zero. Cantor sets.
- A.5. When is $\lim_{n \rightarrow \infty} \int_a^b f_n \, dx = \int_a^b \lim_{n \rightarrow \infty} f_n \, dx$ for the Riemann integral?

Part B. Abstract Measures

- B.1. Abstract measure spaces and integral properties of measurable functions
- B.2. Notions of convergences. Convergence in measure, almost everywhere convergence, etc.
- B.3. Convergence theorems, Fatou's Lemma
- B.4. L^p - spaces, Hölder and Cauchy-Schwartz inequalities. Banach and Hilbert spaces.
- B.5. The Fubini-Tonelli Theorem

Part C. The Lebesgue Measure in $\mathbb{R}^n$

- C.1. The construction of the Lebesgue measure and integral in $\mathbb{R}^n$
- C.2. When is $\lim_{n \rightarrow \infty} \int f_n \, dx = \int \lim_{n \rightarrow \infty} f_n \, dx$ for the Lebesgue integral?
- C.3. The Fubini-Tonelli Theorem revisited
- C.4. Applications: Convolutions and approximations of the identity;
- C.5. The Fourier transform and its inverse. The Fourier transform of L^2 functions

Part D. Differentiation of functions

- D.1. The Vitali Covering Theorem
- D.2. Functions of bounded variation, differentiation of monotone functions, absolute continuity, the Helly Selection Theorem
- D.3. The Lebesgue Differentiation Theorem

References

- E.1. I will type my own course notes which will be posted in Brightspace.
- E.2. I will also post my handwritten lecture notes in Brightspace.
- E.3. There will be no textbook. References: A. Torchinsky, *Real Variables* and W. Rudin, *Real and Complex Analysis*.

MA 54600 Introduction to Functional Analysis

Instructor: Professor Thomas Sinclair

Course Number: MA 54600

Credits: Three

Meeting Time: 4:30–5:45 PM Tuesday / Thursday

Description

The course will primarily focus on covering the basics of functional analysis: Hilbert Spaces, Linear Operators on Hilbert Space, Banach Spaces, Weak Topologies, Banach Algebras, and Spectral Theory. This will follow Conway, Chapters I–VII.

Over the course of the semester, we will plan on taking a few “side quests” such as applications to probability, harmonic analysis, and ergodic theory. Some potential topics, chosen by class interest, are: Reproducing Kernel Hilbert Spaces and Gaussian Processes; Random Walks and Harmonic Functions; Concentration of Measure; Grothendieck’s Inequality and Semi-Definite Programming; and the Pointwise Ergodic Theorem.

There will be regular homework assignments and a few short oral exams. Grades will be determined by the exams and homework.

Required text

J. B. Conway, *A Course in Functional Analysis*, Second Edition, Graduate Texts in Mathematics 96, Springer, 1990.

MA 55300 Introduction To Abstract Algebra

Instructor: Professor Ilia Shkredov

Course Number: MA 55300

Credits: Three

Meeting Time: 11:30–12:20 PM Monday / Wednesday / Friday

Catalog Description

Group theory: Sylow theorems, Jordan–Hölder theorem, solvable groups. Ring theory: unique factorization in polynomial rings and principal ideal domains. Field theory: ruler and compass constructions, roots of unity, finite fields, Galois theory, solvability of equations by radicals.

MA 55400 Linear Algebra

Instructor: Professor Daniel Le

Course Number: MA 55400

Credits: Three

Meeting Time: 8:30–9:20 AM Monday / Wednesday / Friday

Catalog Description

Review of basics: vector spaces, dimension, linear maps, matrices, determinants, linear equations. Bilinear forms; inner product spaces; spectral theory; eigenvalues. Modules over a principal ideal domain; finitely generated abelian groups; Jordan and rational canonical forms for a linear transformation.

MA 55800 Commutative Algebra II

Instructor: Professor Shubhodip Mondal

Course Number: MA 55800

Credits: Three

Meeting Time: 1:30–2:45 PM Tuesday / Thursday

Description

This course provides an introduction to commutative algebra, serving as the foundational language for modern algebraic geometry and algebraic number theory. The course will follow the text by Atiyah and Macdonald.

MA 57200 Introduction to Algebraic Topology

Instructor: Professor Ralph Kaufmann

Course Number: MA 57200

Credits: Three

Meeting Time: 11:30–12:20 PM Monday / Wednesday / Friday

Description

The course is an introduction to algebraic topology. The focus will be on homology and cohomology theory which are basic tools in many subjects.

They are fundamental for topology, but also important in many other fields, such as differential, symplectic and algebraic geometry, number theory, mathematical physics, data science and other applications.

We will treat the classical simplicial and singular homology and cohomology, but we also plan to cover CW complexes and differential forms and more advanced topics, such as homotopy theory, manifolds and relevant aspects of homological algebra as time permits.

The basic text for the course will be *Elements of Algebraic Topology* by James R. Munkres with additions from other sources and the lecture to update the material to a more modern presentation. These will be made available.

MA 57500 Graph Theory

Instructor: Professor Giulio Caviglia

Course Number: MA 57500

Credits: Three

Meeting Time: 10:30–11:45 AM Tuesday / Thursday

Catalog Description

Introduction to graph theory with applications.

MA 59500ANT Introduction to Analytic Number Theory

Instructor: Professor Alisa Sedunova

Course Number: MA 59500ANT

Credits: Three

Meeting Time: 4:30–5:45 PM Tuesday / Thursday

Description

This course explores the distribution of prime numbers, beginning with Euclid’s fundamental proof of the infinitude of primes and progressing to modern estimates for the density of primes up to a given limit.

A central focus of the course is the Prime Number Theorem, which establishes that as numbers grow larger, primes become predictably less frequent and provides an asymptotic formula for the number of primes up to a given point. First proved independently in 1896 by Jacques Hadamard and Charles Jean de la Vallée Poussin, this important result relied heavily on complex analysis and the properties of the Riemann zeta function. Building on Bernhard Riemann’s pioneering 1856 work, we will examine how the distribution of primes connects to the famous Riemann Hypothesis and the zero-free regions of the zeta function.

We will then extend these methods to study primes in arithmetic progressions, demonstrating how prime numbers are distributed across specific sequences. Finally, if time permits we will cover the Bombieri–Vinogradov Theorem from the 1960s, a breakthrough result showing that primes are well-distributed in arithmetic progressions on average across moduli up to a given limit. The proof introduces essential modern techniques, including sieve methods, the large sieve, and Vinogradov’s type I and type II sum decompositions.

Prerequisites

Complex Analysis

MA 59500MDL Mathematical Foundations of Deep Learning

Instructor: Professor Rongjie Lai

Course Number: MA 59500MDL

Credits: Three

Meeting Time: 9:30–10:20 AM Monday / Wednesday / Friday

Description

Deep learning has transformed science, engineering, and many other fields, while raising fascinating mathematical questions: Why can neural networks approximate complex functions? How do they generalize beyond training data? How can they discover low-dimensional structure, learn probability distributions, and adapt to new tasks from context?

This graduate course develops the mathematical foundations needed to study these questions. It is organized around four connected themes: learning functions from data, learning representations of complex data, learning probability laws through generative modeling, and learning solution operators through in-context learning. Along the way, students will encounter ideas from approximation and generalization theory, latent representations, optimal transport, probability flow dynamics, generative models, and operator learning. The course will prepare mathematics graduate students to engage with current research at the intersection of mathematics and deep learning.

MA 59500NT Introduction to Number Theory

Instructor: Professor Trevor Wooley

Course Number: MA 59500NT / MA 49500NT

Credits: Three

Meeting Time: 4:30–5:45 PM Tuesday / Thursday

Content

We begin with a reasonably brisk discussion of the basic notions: the Euclidean algorithm, primes and unique factorisation, congruences, Chinese Remainder Theorem (Public Key Cryptosystems), primitive roots, quadratic reciprocity, arithmetic and multiplicative functions. The second part of the course is devoted to topics: binary quadratic forms, Diophantine approximation and transcendence, continued fractions, Pell's equation and other Diophantine equations, and quadratic fields (subject to time constraints).

Companion Text

An Introduction to the Theory of Numbers (Niven, Zuckerman and Montgomery, 5th edition, Wiley, 1991.)

The course will be based on the instructor's comprehensive web-page hosted LaTeXed notes. Depending on ADA compliance issues, it may be necessary to circulate these notes on paper after April 26th, 2027.

Assessment

Course credit will be based on weekly homeworks – the top 10 scores are totalled; two mid-terms and final exam.

Prerequisites

This course is intended for third- or fourth-year undergraduate students or beginning graduate students who have taken and obtained a grade of B- or better in MA 35301 (Linear Algebra II). Students should have basic competence in mathematical proof.

Number Theory studies the properties of integers, and includes the theory of prime numbers, the arithmetic structures that underlie cryptosystems such as RSA, Diophantine equations (polynomial equations to be solved in integers, including the topic of Fermat's Last Theorem), and rational approximations that distinguish algebraic and transcendental numbers. Although a topic studied for more than two millenia, it is the subject of intense active current research, and connects with many other areas of Mathematics.

This course serves as an introductory exploration of Number Theory, without an abstract algebra prerequisite, so that final year students without a pure mathematics background will find this accessible. Connections with abstract algebra will, however, be noted for interested students, and the material should provide reinforcement and preparation for abstract algebra for those with ambitions in this direction.

MA 59500PGR p -adic Galois Representations

Instructor: Professor Tong Liu

Course Number: MA 59500PGR

Credits: Three

Meeting Time: 9:30–10:20 AM Monday / Wednesday / Friday

Description

We will cover the basic theory of p -adic Galois representations and p -adic Hodge theory. We use Ouyang and Fontaine's book: Theory of p -adic Galois Representations as the main reference. We start with reviewing Galois group of local field and Witt vectors, and then discuss ℓ -adic representations, including Grothendieck's monodromy theorem and Weil–Deligne representation. We will introduce the general idea of admissible representation and provide construction some basic period rings, like A_{inf} , A_{cris} , B_{dR} etc. This will lead the discussion of following important types representations: Hodge–Tate, de Rham, semi-stable and crystalline. We will end with Fontaine–Colmez's theorem: The semi-stable representations are classified by weakly admissible filtered φ -modules. If time allows then I will briefly report the progress of p -adic Hodge theory in the recent years.

MA 59500RM Introduction to Random Matrices

Instructor: Professor Christopher Janjigian

Course Number: MA 59500RM

Credits: Three

Meeting Time: 12:00–1:15 PM Tuesday / Thursday

Description

An introductory graduate course on the spectral behavior of large random matrices, with connections to quantum mechanics and high-dimensional statistics. Topics will include the semicircle and Marchenko–Pastur laws, Gaussian invariant ensembles, an introduction to beta ensembles, bulk and edge scaling limits, the BBP phase transition, and an introduction to universality. Examples will introduce connections to quantum chaos and principal component analysis.

The course will combine proofs of selected foundational results with surveys of more advanced material.

Prerequisites

Familiarity with linear algebra and measure-theoretic probability. Some basic stochastic calculus would be helpful, but is not required. No prior background in quantum mechanics or statistics is assumed.

Intended audience

Graduate students in mathematics and related fields.

MA 59500STIP Scattering Theory and Inverse Problems

Instructor: Professor Plamen Stefanov

Course Number: MA 59500STIP

Credits: Three

Meeting Time: 1:30–2:45 PM Tuesday / Thursday

Description

This course will introduce students to classical scattering theory in Euclidean space, where a “free” system is compared with a “perturbed” one, typically with a compactly supported perturbation, at large times and/or large distances. A classical example is the wave equation $u_{tt} - \Delta u + V(x)u = 0$, with a compactly supported potential $V(x)$, compared with the free wave equation corresponding to $V = 0$. Its stationary counterpart is the Helmholtz equation $(-\Delta + V(x) - k^2)v = 0$. Other examples include scattering by obstacles, by perturbations of the metric, and by time-dependent perturbations.

We will introduce the scattering operator and the scattering amplitude (or scattering kernel), and study their basic properties. Inverse scattering problems ask whether one can recover the potential, obstacle, metric, or other perturbation from the scattering data. In particular, we will see how certain inverse scattering problems can be reduced to inverse boundary value problems on bounded domains and study methods for solving such problems.

Scattering theory is a natural mathematical framework for quantum mechanics, where interactions are often observed indirectly through the behavior of waves or particles far from the scattering region. It also has applications to wave propagation, acoustics, electromagnetics, geophysics, and imaging.

The course will be based on lecture notes already posted on the instructor’s webpage, which will be expanded during the semester. A background in PDEs and basic functional analysis is recommended.

MA 59500TMT Topics in Model Theory

Instructor: Professor Margaret Thomas

Course Number: MA 59500TMT

Credits: Three

Meeting Time: 12:30–1:20 PM Monday / Wednesday / Friday

Description

This course serves as an introduction to model theory (a branch of mathematical logic), which is the study of mathematical structures in terms of their logical properties. Some of the central questions that one can investigate from this perspective include the following. Given a mathematical structure, such as the real or complex field (often enriched with additional operations, such as exponentiation), one can analyze its class of definable sets (that is, the sets that can be defined in that structure using particular kinds of logical expressions). In particular, one can study the algebraic/geometric/combinatorial nature of these definable sets in terms of the logical complexity of their defining expressions, or seek to understand whether or not certain interesting sets/functions can be realised as definable sets therein. Alternatively, one can study the class of all structures in which certain logical expressions are true, considering, for example, how many such structures there can be of any given size, up to isomorphism. By bringing a foundational perspective to core mathematical ideas, model theory is applied to many other areas of mathematics (including algebra, combinatorics, algebraic geometry, number theory, operator theory, dynamical systems, learning theory, ...) and beyond.

The goal of this course is to cover a variety of central concepts in model theory, in particular motivated by various areas of application. Such concepts could include definable sets, elementary extensions, ultraproducts, complete theories, categoricity, model completeness, quantifier elimination, elimination of imaginaries, types, saturated and homogeneous models, and stability. The aim will also be to discuss some of the key examples of model-theoretic structures and their properties, which could include algebraically closed fields (and more generally strongly minimal and stable theories) and real closed fields (and more generally o-minimal and NIP theories), with a view to modern applications of model theory (including applications of o-minimal structures to diophantine geometry, Hodge theory, and dynamical systems; and the role of NIP and related tame structures in combinatorics).

Prerequisites

Key background from a first course in logic (such as MA 58500) will be assumed (including, but not necessarily limited to: first-order formulae, structures and definable sets; the completeness theorem and compactness theorem of first-order logic; and some set theory background such as infinite cardinalities). Certain concepts from abstract algebra will also be helpful as these will be used to illustrate certain key ideas.

MA 61500 Numerical Methods for Partial Differential Equations I

Instructor: Professor Di Qi

Course Number: MA 61500

Credits: Three

Meeting Time: 9:30–10:20 AM Monday / Wednesday / Friday

Description

This is an introductory course on the numerical solution of partial differential equations, designed for graduate students in mathematics, science, and engineering who are interested in computational mathematics. The emphasis is on breadth rather than depth: the course surveys the central ideas of the field while maintaining a balance between rigorous analysis and practical implementation. Topics include the consistency, stability, and convergence of finite difference and finite volume methods for time-dependent problems, including wave equations, parabolic equations, and hyperbolic conservation laws, together with an introduction to the finite element method for elliptic equations. The course also introduces numerical methods for stochastic PDEs, covering the discretization of equations driven by random noise and the notions of strong and weak convergence. We will also discuss iterative solvers for the resulting linear systems, such as the conjugate gradient and multigrid methods, and time integrators for ODEs, such as Runge–Kutta methods and exponential integrators.

Recommended background

Includes linear partial differential equations, linear algebra, and Fourier analysis; the necessary material will be reviewed in lecture.

MA 66100 Modern Differential Geometry

Instructor: Professor Lyuzhou Chen

Course Number: MA 66100

Credits: Three

Meeting Time: 3:30–4:20 PM Monday / Wednesday / Friday

Description

We will cover some standard materials in Riemannian geometry. This includes the notion of Riemannian metric (allowing us to measure lengths and angles of curves) on differentiable

manifolds, connections, geodesics, curvatures, parallel transportation, etc. We will discuss how geometry (especially curvature) interacts with the topology of the manifold. Along the way, as key examples, we will discuss Riemannian manifolds with nice symmetries, for which we can understand the geometry based on the isometry group. This includes hyperbolic geometry and possibly other locally symmetric spaces.

Prerequisites

- (1) MA 562, especially familiarity with differential manifolds, tangent space, vector fields, differential forms;
- (2) some materials from MA 571 and 572, such as compactness, completeness, covering spaces, and fundamental groups.

MA 66500 Algebraic Geometry II

Instructor: Professor Donu Arapura

Course Number: MA 66500

Credits: Three

Meeting Time: 12:00–1:15 PM Tuesday / Thursday

Description

I will assume that everyone has had some prior exposure to affine and projective varieties over an algebraically closed field, either through MA 565 or from something similar. However, I will try to review the basics quickly before moving forward. The ultimate goal of this class is to introduce sheaf theoretic methods into algebraic geometry. Topics would include abstract varieties à la Serre, schemes, complex manifolds, divisors, coherent sheaves and vector bundles, sheaf cohomology, duality and Riemann-Roch for curves.

I won't follow any book very closely, but some of the standard textbooks in this area are listed below. I will also write up lecture notes as I go and make them available,

1. Hartshorne, Algebraic geometry
2. Mumford, Red book of varieties and schemes
3. Shafarevich, Basic algebraic geometry

MA 69300NPDE Analysis and Nonlinear PDEs in Calculus of Variations and Geometric Evolutions

Instructor: Professor Changyou Wang

Course Number: MA 69300NPDE

Credits: Three

Meeting Time: 10:30–11:20 AM Monday / Wednesday / Friday

Description

This course contains:

- The theory of vector-valued Calculus of Variations, which includes discussions on various convexity conditions, especially, the Quasi-Convexity introduced by C. B. Morrey and the Poly-Convexity by John Ball, and their applications.
- The Analysis of the regularity and the structures of singular sets of harmonic maps between manifolds, as well as equilibrium configurations of the Oseen-Frank model of liquid crystals.
- Heat flow of harmonic maps between manifolds, that is, the gradient flow of the Dirichlet energy functional of mappings between manifolds. It includes the issues of both the existence and the uniqueness, the regularity of suitable weak solutions in dimensions two and higher.

The course will provide a list of suggested homework/potential research problems. It will be strongly encouraged (not required) that students may choose a problem and form a small group to collectively work on it, under the supervision of the instructor.

The course will conclude with a final class presentation.

Prerequisites

Some knowledge on linear PDEs and real analysis will be helpful, but not absolutely necessary.

MA 69600AV Abelian Varieties

Instructor: Professor Shubhodip Mondal

Course Number: MA 69600AV

Credits: Three

Meeting Time: 12:00–1:15 PM Tuesday / Thursday

Description

This course gives a rigorous introduction to abelian varieties following Mumford's *Abelian Varieties*, beginning with the general theory of abelian varieties over an algebraically closed field, including the theorem of the cube, line bundles, and the construction of the dual abelian variety. We then develop the theory of isogenies, the Picard scheme, and polarizations, culminating in the analytic theory over $\mathbb{C}$ via complex tori and theta functions. Subsequent weeks cover the Néron–Severi group, endomorphism algebras (Poincaré reducibility, Rosati involution), and the structure of the Tate module together with the Weil pairing. We will discuss the moduli-theoretic perspective, including a treatment of the Torelli theorem and a brief look at Jacobians of curves as a source of examples. Time permitting, the course closes with an introduction to abelian schemes over general bases and their relevance to arithmetic geometry, connecting to topics such as the Tate conjecture.

Prerequisites

First course in algebraic geometry (varieties, schemes, and coherent sheaves) and familiarity with basic commutative algebra.