1) Solve the system

$$\begin{aligned} 2x + y + z &= 5 \\ x - y + z &= 0 \\ 3x \quad\;\; - z &= 1 \end{aligned}$$

2) Find a so that a solution exists to the system

$$\begin{aligned} 2x \;\; +y &= 5 \\ x \;\; -y &= 1 \\ 3x \;\; -y &= a \end{aligned}$$

3) Let

$$A = \begin{bmatrix} 2 & -3 & 2 \\ 1 & 0 & 0 \\ 0 & 2 & 1 \end{bmatrix}$$

a) Invert this matrix using elementary row operations.
b) Invert this matrix using the cofactor expansion.

4) If A is a 3×3 matrix with $\det(A) = 2$
a) Find $\det(A^{-1})$.
b) Find $\det(A^T)$.
c) Find $\det(5A)$.

5) True or False
a) $\det(A) = \det(P^{-1}AP)$.
b) If $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ then $\det(A) = 1$.
c) If $\text{rref}(A) = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix}$ then $\det(A) = 0$.

6) Using the cofactor expansion, compute

$$\det \begin{bmatrix} 1 & -1 & -1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 1 & 0 & 0 \\ 0 & -1 & 0 & 2 \end{bmatrix}.$$

7) Find the vector which has three times the magnitude of $\vec{u} = (2, -1, 3)$ and points in the oppsite direction of $\vec{v} = (1, -2, 4)$.

8) Which of the following functions from $\mathbb{R}^3$ to $\mathbb{R}^2$ are linear transformtions.

a) $L(x, y, z) = (2x + y, 3y - z)$.

b) $L(x, y, z) = (x + yz, z)$.

c) $L(x, y, z) = (x - y + z, 2x - 1)$.

d) $L(x, y, z) = (0, 2x + y - 3z)$.

9) If L is the linear transformation from $\mathbb{R}^2$ to $\mathbb{R}^2$ obtained by rotating the plane by $\pi/3$ radians and then reflecting across the x-axis, then find the representation of L with respect to the standard basis.

10) Find a parametric equation for the line of intersection between the planes $x - 3y + 5z = 2$ and $2x + y - 4z = 5$.

11) Which of the following is a vector space.

a) The set of all polynomials $ax^2 + bx + c$ with $a = -b$ (as a subset of P^2).

b) The set of all (a, b, c) with $ab = c$ (as a subset of $\mathbb{R}^3$).

c) The null space of the matrix $\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 1 \end{bmatrix}$ (as a subset of $\mathbb{R}^3$).

d) The set of all (a, b, c, d) with $a + b + c + d = 1$ (as a subset of $\mathbb{R}^4$).

12) Is $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ in span $\left\{ \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

If so, write $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ as a linear combination of $\begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}$ and $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$.

If not, find the distance from $\begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$ to span $\left\{ \begin{bmatrix} 2 \\ 2 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix} \right\}$.

13) Find the dimension of

$$\text{span} \left\{ \begin{bmatrix} 1 \\ 0 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix} \right\}$$

14) Let

$$A = \begin{bmatrix} 1 & 1 & 2 & -1 & 1 \\ -1 & -1 & 0 & -1 & 1 \\ 0 & 0 & 2 & -2 & 2 \\ 1 & 1 & 1 & -1 & 1 \end{bmatrix}$$

a) Find a basis for the column space of A.
b) Find a basis for the row space of A.
c) Find a basis for the null space of A.
d) Find a basis for the orthogonal complement of the column space of A.
e) Find a basis for the orthogonal complement of the row space of A.
f) Find the rank of A.
g) Find the nullity of A.

15) Let $T = \{(2,3), (-2,1)\}$ be a basis of $\mathbb{R}^2$ and $S\{(1,1), (0,3)\}$ be another basis of $\mathbb{R}^2$.
a) Find the transition matrix $P_{S \leftarrow T}$ from the basis T to the basis S.
b) If $[\vec{v}]_S = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ find $[\vec{v}]_T$ and $\vec{v}$ in the standard basis.

16) Use Gramm-Schmidt to find an orthonormal basis for

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

17) Find an orthonormal basis for the subspace of $\mathbb{R}^3$ consisting of all (x, y, z) with $2x + y - z = 0$.

18) Let W be the subpace of $\mathbb{R}^4$ defined by

$$\text{span}\left\{ \begin{bmatrix} 1 \\ 1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix} \right\}.$$

Write the vector $\vec{u} = \vec{w} + \vec{v}$ where $\vec{w}$ is a vector in W and $\vec{v}$ is a vector in $W^\perp$ and

$$\vec{u} = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}.$$

19) Find the distance from $(1, 1, 1)$ to the plane $x + y - z = 1$.

20) If A is a 4×4 matrix whose characteristic polynomial is $\lambda^4 - 7\lambda^2 + 12$, find a diagonal matrix D which is similar to A.

21) Find the eigenvalues and eigenvectors of

$$A = \begin{bmatrix} 2 & 2 & 0 \\ 0 & 3 & 1 \\ 0 & 0 & 4 \end{bmatrix}.$$

Find a matrix P and diagonal matrix D, such that $P^{-1}AP = D$.

22) Find an orthognal matrix P so that $P^{-1}AP$ is diagonal, for

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 2 & 1 \\ 1 & 1 & 2 \end{bmatrix}.$$

23) Find an orthognal matrix P so that $P^{-1}AP$ is diagonal, for

$$A = \begin{bmatrix} 1 & 2 \\ 2 & 4 \end{bmatrix}.$$

24) Find the least squares approximation solution to the matrix equation

$$\begin{bmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \\ 1 & -1 \end{bmatrix} \begin{bmatrix} a \\ b \end{bmatrix} = \begin{bmatrix} 2 \\ 3 \\ 4 \\ 5 \end{bmatrix}.$$

25) Find the least squares line which approximates the points $(1,1)$, $(2,2)$, $(3,2)$, $(4,3)$, and $(5,4)$.

26) Find the general solution to

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 1 & -1 \\ -3 & 3 \end{bmatrix} \vec{x}.$$

27) Find the solution to

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 4 & -2 \\ 1 & 1 \end{bmatrix} \vec{x} \quad \text{with } \vec{x}(0) = \begin{bmatrix} 1 \\ 0 \end{bmatrix}$$

28) Find the general real solution to

$$\frac{d\vec{x}}{dt} = \begin{bmatrix} 2 & 2 \\ -2 & 2 \end{bmatrix} \vec{x}.$$