In problems 1-4, find the general solution to the differential equation.

1. $\mathbf{x}'(t) = \begin{bmatrix} 1 & 1 \\ -1 & 1 \end{bmatrix} \mathbf{x}(t)$

2. $\mathbf{x}'(t) = \begin{bmatrix} 2 & -1 \\ 1 & 2 \end{bmatrix} \mathbf{x}(t)$

3. $\mathbf{x}'(t) = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix} \mathbf{x}(t)$

4. $\mathbf{x}'(t) = \begin{bmatrix} 3 & 5 \\ -1 & 1 \end{bmatrix} \mathbf{x}(t)$

5. Find the particular solution $\mathbf{x}(t)$ to the differential equation of problem 1 such that
$\mathbf{x}(0) = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$.

6. Find the particular solution $\mathbf{x}(t)$ to the differential equation of problem 2 such that
$\mathbf{x}(0) = \begin{bmatrix} -1 \\ 2 \end{bmatrix}$.

7. Find the particular solution $\mathbf{x}(t)$ to the differential equation of problem 3 such that
$\mathbf{x}(0) = \begin{bmatrix} 4 \\ -3 \end{bmatrix}$.

8. Find the particular solution $\mathbf{x}(t)$ to the differential equation of problem 4 such that
$\mathbf{x}(0) = \begin{bmatrix} 5 \\ 1 \end{bmatrix}$.

Answers

1. $\mathbf{x}(t) = c_1 e^t \begin{bmatrix} \sin t \\ \cos t \end{bmatrix} + c_2 e^t \begin{bmatrix} -\cos t \\ \sin t \end{bmatrix}$.

3. $\mathbf{x}(t) = c_1 e^{3t} \begin{bmatrix} -\cos(2t) - \sin(2t) \\ 2\cos(2t) \end{bmatrix} + c_2 e^{3t} \begin{bmatrix} \cos(2t) - \sin(2t) \\ 2\sin(2t) \end{bmatrix}$.

5. $\mathbf{x}(t) = e^t \begin{bmatrix} \cos t + 2 \sin t \\ 2 \cos t - \sin t \end{bmatrix}.$

7. $\mathbf{x}(t) = e^{3t} \begin{bmatrix} 4 \cos(2t) - \sin(2t) \\ -3 \cos(2t) + 5 \sin(2t) \end{bmatrix}.$