1. Let $A = \begin{bmatrix} 1 & 2 & 0 \\ 1 & 1 & 2 \\ 0 & 1 & 2 \end{bmatrix}$ and $B = \begin{bmatrix} a \\ 0 \\ 0 \end{bmatrix}$. Find all real numbers a such that $B^\top A = (AB)^\top$.

A. $a \neq 0$

B. $a = \pm 1$

C. $a = 1$

D. $a = -1$

E. $a = 0$

2. Find all values of a for which the following system of equations has no solution.

$$\begin{array}{ccccccc} x & + & y & - & z & = 2 \\ x & + & 2y & + & z & = 3 \\ x & + & y & + & (a^2-5)z & = a \end{array}$$

A. $a = 1$

B. $a = \pm 2$

C. $a = 2$

D. $a = -2$

E. $a = \pm 4$

3. If $A = \begin{bmatrix} 0 & 1 & 0 \\ 2 & 0 & -1 \\ 1 & 0 & -1 \end{bmatrix}$, then A^{-1} has the form

$$A^{-1} = \begin{bmatrix} a & 1 & -1 \\ 1 & b & 0 \\ 0 & 1 & c \end{bmatrix}.$$

Find the sum $a + b + c$.

A. 0

B. 1

C. -1

D. 2

E. -2

4. Suppose that $\mathbf{u}$ and $\mathbf{v}$ are unit vectors such that $(\mathbf{u}, \mathbf{v}) = -1/2$. Find $|3\mathbf{u} + \mathbf{v}|$.

A. 2

B. $\sqrt{3}$

C. $\sqrt{5}$

D. $\sqrt{7}$

E. 4

5. Let A be a 5×8 matrix. Which of the following statements are true.

(1) The nullity of A cannot equal to 5.
(2) The rank of A cannot equal 3.
(3) If the nullity of A is 4 then the nullity of $A^\top$ is 1.
(4) If the rank of A is 3, then the nullity of A is 2.

A. (1) and (2).

B. (3) and (4).

C. (3) only.

D. (4) only.

E. None are true.

6. Find a basis for the subspace W of R^3 consisting all vectors $\begin{bmatrix} x \\ y \\ z \end{bmatrix}$ such that $2x + y - z = 0$.

A. $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 1 \\ 1 \\ -3 \end{bmatrix}, \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 3 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 2 \\ 4 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 1 \\ 3 \end{bmatrix}, \begin{bmatrix} -1 \\ 0 \\ -2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$

7. Suppose that the Gram-Schmidt process is applied to a certain set of vectors $\mathbf{v}_1$, $\mathbf{v}_2$, $\mathbf{v}_3$ in R^4 to obtain an orthonormal set of vectors $\mathbf{w}_1$, $\mathbf{w}_2$, $\mathbf{w}_3$. The first two turn out to be

$$\mathbf{w}_1 = (1/\sqrt{2}, -1/\sqrt{2}, 0, 0), \quad \mathbf{w}_2 = (1/2, 1/2, 1/2, 1/2).$$

Given that $\mathbf{v}_3 = (1, 0, 1, -1)$, find $\mathbf{w}_3$.

A. $\mathbf{w}_3 = \frac{1}{2}(1, 1, -1, -1)$

B. $\mathbf{w}_3 = \frac{1}{6}(1, 1, 3, -5)$

C. $\mathbf{w}_3 = \frac{1}{\sqrt{18}}(2, 2, -3, -1)$

D. $\mathbf{w}_3 = \frac{1}{\sqrt{22}}(1, 1, 2, -4)$

E. $\mathbf{w}_3 = \frac{1}{\sqrt{24}}(2, 2, 0, -4)$

8. Which of the given subsets are **NOT** subspaces of R^3.

(1) The set of all vectors of the form $(a, 0, c)$.

(2) The set of all vectors (a, b, c) such that $a - b + c = 0$ and $a + 2c = 0$.

(3) The set of all vectors of the form $(a, 1 - a - c, c)$.

(4) The set all vectors $\mathbf{v}$ such that $\mathbf{v} \cdot \mathbf{a} = \mathbf{v} \cdot \mathbf{b}$, where $\mathbf{a} = (1, 1, 1)$ and $\mathbf{b} = (1, -1, 1)$.

(5) The set of all linear combinations of the vectors $(1, 1, 0)$ and $(1, 2, 1)$.

(6) The set all vectors perpendicular to the vector $(1, 1, 1)$.

A. (3) only.

B. (3) and (4).

C. (4) and (6)

D. (1), (2), and (4)

E. All are subspaces.

9. Let A be a 3×4 matrix such that $\text{rank}(A) = 3$. Which of the following statements are true.

(1) $A\mathbf{x} = \mathbf{b}$ always has a solution

(2) The rows of A are linearly independent.

(3) The columns of A span R^3.

(4) $A\mathbf{x} = \mathbf{0}$ has infinitely many solutions.

(5) The dimension of the orthogonal complement of the row space of A is 1.

A. (2) and (3) only.

B. (2), (3), (4) and (5) only.

C. (1) only.

D. (1), (2), (4) and (5) only.

E. All are true.

10. Find a basis for the nullspace of A, given that

$$A = \begin{bmatrix} 1 & 4 & 5 & -9 & -7 \\ 0 & 3 & 6 & -4 & -9 \\ 1 & 2 & 1 & -3 & -1 \\ 2 & 3 & 0 & -3 & 1 \end{bmatrix} \longrightarrow \begin{bmatrix} 1 & 0 & -3 & 0 & 5 \\ 0 & 1 & 2 & 0 & -3 \\ 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} = \text{rref}(A).$$

A. $\begin{bmatrix} 5 \\ 6 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -7 \\ -9 \\ -1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ 3 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 9 \\ 4 \\ 3 \\ 3 \end{bmatrix}$

C. $\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ 0 \\ 0 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} -3 \\ 2 \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 5 \\ -3 \\ 0 \\ 0 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 3 \\ -2 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -5 \\ 3 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

11. Let W be the subspace of R^4 spanned by $Y_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$ and $Y_2 = \begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$.

Find the orthogonal projection of $X = \begin{bmatrix} 3 \\ -1 \\ -1 \\ 3 \end{bmatrix}$ onto W.

A. $\begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ -1 \\ 1 \\ -1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \\ 1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 1 \\ 1 \\ 1 \end{bmatrix}$

12. Evaluate the determinant of

$$A = \begin{bmatrix} 1 & 2 & -3 & 4 \\ -4 & 2 & 1 & 3 \\ 3 & 0 & 0 & -3 \\ 2 & 0 & -2 & 3 \end{bmatrix}.$$

A. 120

B. 96

C. 64

D. 48

E. 24

13. Given that $\det\begin{bmatrix} a & b & c \\ x & y & z \\ 1 & 1 & 1 \end{bmatrix} = 4$, find

$$\det\begin{bmatrix} x & y & z \\ (x-2a) & (y-2b) & (z-2c) \\ 3 & 3 & 3 \end{bmatrix}.$$

A. 24

B. -24

C. 12

D. -12

E. 6

14. For what values of a is the matrix A singular (not invertible).

$$A = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ a-1 & 2 & 0 & 0 & 0 \\ 0 & 0 & a & 0 & 1 \\ 0 & 0 & 0 & 0 & -2 \\ 0 & 0 & 0 & 3 & 0 \end{bmatrix}$$

A. $a = -3, 1$

B. $a = -1, 0$

C. $a = 0, 1$

D. $a = 0, 3$

E. $a = 1, 3$

15. The matrix $A = \begin{bmatrix} 1 & -1 \\ 2 & 4 \end{bmatrix}$ has eigenvalues 2 and 3. An eigenvector corresponding to 2 is:

A. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

B. $\begin{bmatrix} -1 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} -1 \\ 4 \end{bmatrix}$

E. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

16. Which of the following matrices is diagonalizable.

$$(1)\begin{bmatrix} 2 & 1 & 3 \\ 0 & 0 & 4 \\ 0 & 0 & 1 \end{bmatrix} \quad (2)\begin{bmatrix} 2 & 1 & 3 \\ 0 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \quad (3)\begin{bmatrix} 2 & 1 & 3 \\ 1 & 2 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

A. All of them.

B. None of them.

C. (3) only.

D. (1) and (3) only.

E. (2) and (3) only.

17. Find the particular solution to the differential equation

$$\begin{bmatrix} x_1' \\ x_2' \end{bmatrix} = A \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \text{where} \quad A = \begin{bmatrix} 3 & 2 \\ 2 & 3 \end{bmatrix},$$

such that $x_1(0) = 3$ and $x_2(0) = -1$. The eigenvalues of A are 1 and 5 with corresponding eigenvectors $P_1 = \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ and $P_2 = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$.

A. $\begin{bmatrix} 2e^t + e^{5t} \\ -2e^t + e^{5t} \end{bmatrix}$

B. $\begin{bmatrix} 2e^t + e^{5t} \\ 2e^t - 3e^{5t} \end{bmatrix}$

C. $\begin{bmatrix} e^t + 2e^{5t} \\ e^t - 2e^{5t} \end{bmatrix}$

D. $\begin{bmatrix} e^t + 2e^{5t} \\ 2e^t - 3e^{5t} \end{bmatrix}$

E. $\begin{bmatrix} 4e^t - e^{5t} \\ -4e^t + 3e^{5t} \end{bmatrix}$

18. Find all values of a such that $\begin{bmatrix} 1 \\ 0 \\ a \end{bmatrix}$ is an eigenvector for the matrix

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}.$$

A. $a = 2,\ a = 0$

B. $a = 1$

C. $a = 1,\ a = -1$

D. $a = 2,\ a = -3$

E. $a = 2$

19. A 3×3 matrix A has eigenvalues $\lambda = 1, 2$ and the eigenspace of $\lambda = 1$ is spanned by

$$P_1 = \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \quad P_2 = \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}.$$

Given that A is symmetric, find an eigenvector for $\lambda = 2$.

A. $\begin{bmatrix} 1 \\ -1 \\ -1 \end{bmatrix}$

B. $\begin{bmatrix} 2 \\ 2 \\ 0 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 2 \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} 2 \\ -1 \\ -1 \end{bmatrix}$

E. $\begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$

20. Determine the least squares solution to $A\mathbf{x} = \mathbf{b}$, where

$$A = \begin{bmatrix} 1 & 1 \\ 1 & 1 \\ 1 & -1 \end{bmatrix} \quad \text{and} \quad \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}.$$

A. $\begin{bmatrix} 0 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 1/4 \\ 1/4 \end{bmatrix}$

C. $\begin{bmatrix} 1/3 \\ 1/3 \end{bmatrix}$

D. $\begin{bmatrix} 1/2 \\ 1/2 \end{bmatrix}$

E. $\begin{bmatrix} 2/3 \\ 2/3 \end{bmatrix}$

21. Find x given that

$$\begin{array}{ccccccc} 2x & + & ay & + & 3z & = & a \\ 5x & + & by & - & z & = & b \\ x & + & cy & + & z & = & c \end{array}$$

and

$$\det \begin{bmatrix} 2 & a & 3 \\ 5 & b & -1 \\ 1 & c & 1 \end{bmatrix} \neq 0.$$

A. $-b + 17c - 6a$

B. $5/(-b + 17c - 6a$

C. 0

D. 1

E. $2b - 5a$

22. Let A be a 6×3 matrix whose null space is spanned by

$$\begin{bmatrix} 1 \\ 2 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 3 \\ 3 \\ 0 \end{bmatrix}$$

What is the rank of A.

A. 1

B. 2

C. 3

D. 4

E. 5

23. Let A be a 3×1 matrix, B a 2×3 matrix and C a 2×1 matrix. Which of the following is defined and is a 1×2 matrix.

A. AB

B. $B^\top C$

C. $C^\top BA$

D. $BAC^\top$

E. $(BA)^\top$

24. For what value of k is the vector $\mathbf{w} = (1, 6, k)$ in the subspace spanned by the vectors

$$\mathbf{w}_1 = (1, 2, 3), \quad \mathbf{w}_2 = (1, -2, 3), \quad \mathbf{w}_3 = (4, 4, 12).$$

A. 1

B. 2

C. 3

D. 4

E. 5

25. Find the dimension of the subspace of M_{22} consisting of matrices

$$\begin{bmatrix} a & b \\ c & d \end{bmatrix}$$

such that

$$\begin{array}{rcrcrcrcl} a & + & 2b & + & 3c & + & 4d & = & 0 \\ 3a & + & 6b & + & 9c & + & 12d & = & 0 \end{array}$$

A. 0

B. 1

C. 2

D. 3

E. 4

26. Let A be a nonsingular $n \times n$ matrix. Which statement is false.

A. 0 is an eigenvalue of A.

B. $\det(A) \neq 0$.

C. The rank of A is n.

D. $A^\top$ is nonsingular.

E. A^2 is nonsingular.

27. Let V be an inner product space, and let $\mathbf{u}$ and $\mathbf{v}$ be unit vectors such that $(\mathbf{u}, \mathbf{v}) = 0$. Find $(3\mathbf{u} - 4\mathbf{v}, \mathbf{u} + 5\mathbf{v})$.

A. 17

B. -17

C. 3

D. 20

E. -60

28. Let a be a real number such that $a \neq -1, 0, 1$ and let

$$A = \begin{bmatrix} 1 & a \\ a & 1 \end{bmatrix}.$$

Which of the following vectors is an eigenvalue for A.

A. $\begin{bmatrix} 1 \\ 0 \end{bmatrix}$

B. $\begin{bmatrix} 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} 1 \\ 1 \end{bmatrix}$

D. $\begin{bmatrix} 1 \\ a \end{bmatrix}$

E. $\begin{bmatrix} a \\ 1 \end{bmatrix}$

29. Which of the following transformations from R^3 to R^2 is **not** linear.

A. $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} x \\ y \end{bmatrix}$

B. $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2x \\ 0 \end{bmatrix}$

C. $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} xy \\ x+y \end{bmatrix}$

D. $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 2y \\ x+y \end{bmatrix}$

E. $L \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{bmatrix} \begin{bmatrix} x \\ y \\ x \end{bmatrix}$

30. Let $A = \begin{bmatrix} 0 & i \\ -i & 0 \end{bmatrix}$. Then A^5 is equal to

A. I

B. A^2

C. $-A^2$

D. A

E. $-A$

31. Suppose that A and B are 3×3 matrices such that $\det(A) = 9$ and $B^2 = A$. Find the value of $\det(2AB^\top A^{-1}B)$.

A. 18

B. 72

C. 2

D. 8

E. 162

32. For what value of t does the system with the following augmented matrix have no solution.

$$\begin{bmatrix} 0 & t-7 & 0 & | & 6 \\ 0 & 2 & 2t-2 & | & -2 \\ 1 & -1 & -2 & | & 1 \end{bmatrix}$$

A. 1

B. 7

C. 9

D. -1

E. -9

33. For what value of t does the system with the following augmented matrix have an infinite number of solutions.

$$\begin{bmatrix} 0 & t-7 & 0 & | & 6 \\ 0 & 2 & 2t-2 & | & -2 \\ 1 & -1 & -2 & | & 1 \end{bmatrix}$$

A. 1

B. 7

C. 9

D. -1

E. -9

34. Let W be the subspace of R^3 spanned by $(1,0,2)$ and $(0,1,0)$. Let $\mathbf{v} = (1,2,3)$, and let $\mathbf{v}'$, $\mathbf{v}''$ be such that $\mathbf{v} = \mathbf{v}' + \mathbf{v}''$, where $\mathbf{v}'$ is in W and $\mathbf{v}''$ is in $W^{\perp}$. Find the vector $\mathbf{v}'$.

A. $\mathbf{v}' = \frac{1}{5}(7, 10, 14)$

B. $\mathbf{v}' = (3, 1, 4)$

C. $\mathbf{v}' = \frac{1}{5}(3, 10, -6)$

D. $\mathbf{v}' = \frac{1}{3}(3, 1, 9)$

E. $\mathbf{v}' = \frac{1}{5}(3, 10, 4)$

35. Suppose that the system of equations

$$\begin{array}{rcrcrcl} x & + & 2y & - & 3z & = & a \\ 2x & + & 3y & + & 3z & = & b \\ 5x & + & 9y & - & 6z & = & c \end{array}$$

has a solution. Which equation must a, b, and c satisfy.

A. $a + b + c = 0$

B. $a + 2b - 3c = 0$

C. $3a + b - c = 0$

D. $3a - 2b + c = 0$

E. $a - b + 3c = 0$

36. Lrt $L : R^3 \to R^3$ be the linear transformation such that

$$L\left(\begin{bmatrix}1\\0\\0\end{bmatrix}\right) = \begin{bmatrix}2\\-1\\1\end{bmatrix}, \quad L\left(\begin{bmatrix}0\\1\\0\end{bmatrix}\right) = \begin{bmatrix}1\\3\\2\end{bmatrix}, \quad L\left(\begin{bmatrix}0\\0\\1\end{bmatrix}\right) = \begin{bmatrix}1\\4\\2\end{bmatrix}.$$

Find $L\left(\begin{bmatrix}1\\-1\\1\end{bmatrix}\right)$.

A. $\begin{bmatrix}1\\3\\1\end{bmatrix}$

B. $\begin{bmatrix}1\\1\\1\end{bmatrix}$

C. $\begin{bmatrix}4\\6\\3\end{bmatrix}$

D. $\begin{bmatrix}-2\\2\\1\end{bmatrix}$

E. $\begin{bmatrix}2\\0\\1\end{bmatrix}$

37. Let $A = \begin{bmatrix}3 & 5\\2 & 4\end{bmatrix}$. If $A^{-1} = \begin{bmatrix}a & b\\c & d\end{bmatrix}$, find $a + d$.

A. $-3/2$

B. -2

C. 1

D. $7/2$

E. $13/2$

38. Find all value(s) of k such that the vectors

$$(1, 3, k, 1), \quad (1, 1, 0, -1), \quad (0, 1, 1, 1)$$

are linearly dependent.

A. $k = 2$

B. $k = -1, 3$

C. $k \neq 1$

D. $k \neq 2$

E. $k = 3$

39. Find the matrix B such that $\text{adj}(B) = \begin{bmatrix} 2 & 4 \\ -5 & 7 \end{bmatrix}$.

A. $\begin{bmatrix} 7 & -4 \\ 5 & 2 \end{bmatrix}$

B. $\begin{bmatrix} 2 & 4 \\ -5 & 7 \end{bmatrix}$

C. $\begin{bmatrix} 2 & -5 \\ 4 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 7 & -5 \\ 2 & 4 \end{bmatrix}$

E. $\begin{bmatrix} -5 & 7 \\ 2 & 4 \end{bmatrix}$

40. Find a diagonal matrix similar to $A = \begin{bmatrix} 4 & 1 \\ -3 & 8 \end{bmatrix}$.

A. $\begin{bmatrix} 2 & 0 \\ 0 & -4 \end{bmatrix}$

B. $\begin{bmatrix} 4 & 0 \\ 0 & 8 \end{bmatrix}$

C. $\begin{bmatrix} 5 & 0 \\ 0 & 7 \end{bmatrix}$

D. $\begin{bmatrix} 4 & 0 \\ 0 & 9 \end{bmatrix}$

E. $\begin{bmatrix} -1 & 0 \\ 0 & 3 \end{bmatrix}$

41. Let A be a 3×5 matrix of rank 2. Which statement is true.

A. For every $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has a unique solution.

B. For some $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has a unique solution, and for some $\mathbf{b}$, it has no solution.

C. For every $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions.

D. For some $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has an infinite number of solutions, and for some $\mathbf{b}$, it has no solution.

E. For every $\mathbf{b}$, $A\mathbf{x} = \mathbf{b}$ has no solution.

42. Let A, B, and C be 2×2 matrices and k a scalar. Which statements are true.

1. If $A^2 = A$, then $A = I_n$, $-I_n$ or 0.
2. $\det(A + B) = \det(A) + \det(B)$.
3. $\det(kA) = k\det(A)$.
4. If λ is an eigenvalue of A, then λ^2 is an eigenvalue of A^2.
5. $A\,\mathrm{adj}(A) = \det(A)I_2$.

A. 1, 2, and 5.

B. 3, 4, and 5.

C. 1, 2, 3, 4, and 5.

D. 2 and 3

E. 4 and 5.

43. Let $A = \begin{bmatrix} a & b & c \\ x & y & z \\ 3 & 6 & 9 \end{bmatrix}$ Which of the following matrices have the same determinant as A.

1. $\begin{bmatrix} 3a & 3b & 3c \\ x & y & z \\ 1 & 2 & 3 \end{bmatrix}$
2. $\begin{bmatrix} a-x & b-y & c-z \\ x & y & z \\ 3 & 6 & 9 \end{bmatrix}$
3. $\begin{bmatrix} x & y & z \\ a & b & c \\ 3 & 6 & 9 \end{bmatrix}$
4. $\begin{bmatrix} x & y & z \\ 3 & 6 & 9 \\ a+x+3 & b+y+6 & c+z+9 \end{bmatrix}$

A. 1, 2, and 4.

B. 1 and 2.

C. 2, 3, and 4.

D. 1, 2, 3, and 4.

E. 1 and 4.

44. An eigenvalue of

$$A = \begin{bmatrix} 1 & 0 & 2 \\ 2 & 1 & 1 \\ 3 & 0 & 2 \end{bmatrix}$$

is $\lambda = 4$. Find a basis for its eigenspace.

A. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 4 \end{bmatrix}$

B. $\begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} -2 \\ 2 \\ 5 \end{bmatrix}$

D. $\begin{bmatrix} 5 \\ 9 \\ 3 \end{bmatrix}$

E. $\begin{bmatrix} 6 \\ 7 \\ 9 \end{bmatrix}$

45. The matrix $A = \begin{bmatrix} -1 & -2 \\ 2 & -1 \end{bmatrix}$ has eigenvalues $-1 \pm 2i$, and an eigenvector corresponding to $-1 + 2i$ is $\begin{bmatrix} i \\ 1 \end{bmatrix}$. Find the solution to the differential equation

$$\mathbf{x}'(t) = A\mathbf{x}(t)$$

such that $\mathbf{x}(0) = \begin{bmatrix} -1 \\ 1 \end{bmatrix}$.

A. $\mathbf{x}(t) = e^{-t} \begin{bmatrix} -\sin(2t) - \cos(2t) \\ -\sin(2t) + \cos(2t) \end{bmatrix}$

B. $\mathbf{x}(t) = e^{-t} \begin{bmatrix} 2\sin(2t) - \cos(2t) \\ -2\sin(2t) + \cos(2t) \end{bmatrix}$

C. $\mathbf{x}(t) = e^{2t} \begin{bmatrix} \sin(t) - \cos(t) \\ -\sin(t) + \cos(t) \end{bmatrix}$

D. $\mathbf{x}(t) = e^{2t} \begin{bmatrix} \sin(-t) - \cos(-t) \\ -\sin(-t) + \cos(-t) \end{bmatrix}$

E. $\mathbf{x}(t) = e^{-t} \begin{bmatrix} i\sin(2t) - \cos(2t) \\ 2i\sin(2t) + \cos(2t) \end{bmatrix}$

46. Find the least squares line for the data points

$$(-1,3), \quad (0,2), \quad (1,4).$$

A. $y = \frac{1}{6}x + 3$

B. $y = \frac{1}{3}x + 2$

C. $y = x + 3$

D. $y = \frac{1}{2}x + \frac{5}{2}$

E. $y = \frac{1}{2}x + 3$

47. Find the eigenvalues of $A = \begin{bmatrix} 4 & 5 \\ -1 & 2 \end{bmatrix}$.

A. $4 \pm 2i$

B. $3 \pm 2i$

C. $1 \pm i$

D. $4 \pm 2i$

E. $4 \pm i$

48. Let $A = \begin{bmatrix} 1 & -2 \\ 4 & 5 \end{bmatrix}$. An eigenvalue of A is $\lambda = 3 + 2i$. Find a corresponding eigenvector.

A. $\begin{bmatrix} 1-i \\ 2 \end{bmatrix}$

B. $\begin{bmatrix} 2+i \\ 1 \end{bmatrix}$

C. $\begin{bmatrix} -i \\ 2 \end{bmatrix}$

D. $\begin{bmatrix} -1+i \\ 2 \end{bmatrix}$

E. $\begin{bmatrix} 1+4i \\ 1 \end{bmatrix}$

49. Suppose that A is an orthogonal 3×3 matrix. Then $\det(3A^2) =$.

A. 0

B. 3

C. 1/3

D. 9

E. 27

50. Find the general solution to the differential equation

$$\mathbf{x}'(t) = \begin{bmatrix} 1 & 0 & 4 \\ 0 & 2 & 2 \\ 0 & 0 & 3 \end{bmatrix} \mathbf{x}(t)$$

A. $\mathbf{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 e^{3t} \begin{bmatrix} 2 \\ 2 \\ 1 \end{bmatrix}$

B. $\mathbf{x}(t) = c_1 e^t \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_3 e^{3t} \begin{bmatrix} 1 \\ 0 \\ 1 \end{bmatrix}$

C. $\mathbf{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 0 \\ 2 \\ 0 \end{bmatrix} + c_3 e^{3t} \begin{bmatrix} 4 \\ 2 \\ 3 \end{bmatrix}$

D. $\mathbf{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 0 \\ 2 \\ 2 \end{bmatrix} + c_3 e^{3t} \begin{bmatrix} -2 \\ 1 \\ 1 \end{bmatrix}$

E. $\mathbf{x}(t) = c_1 e^t \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix} + c_2 e^{2t} \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} + c_3 e^{3t} \begin{bmatrix} -4 \\ -2 \\ -3 \end{bmatrix}$

Answers

1.E. **2.**D. **3.**E. **4.**D. **5.**C. **6.**A. **7.**B. **8.**A. **9.**E. **10.**C. **11.**E. **12.**D. **13.**A. **14.**D. **15.**B. **16.**D. **17.**A. **18.**C. **19.**A. **20.**B. **21.**C. **22.**A. **23.**E. **24.**C. **25.**D. **26.**A. **27.**B. **28.**C. **29.**C. **30.**D. **31.**B. **32.**B. **33.**A. **34.**A. **35.**C. **36.**E. **37.**D. **38.**A. **39.**A. **40.**C. **41.**D. **42.**E. **43.**A. **44.**E. **45.**A. **46.**E. **47.**B. **48.**D. **49.**E. **50.**A.