Computer Project # 1

Nonlinear Springs

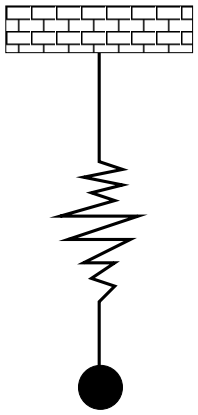
Goal: Investigate the behavior of nonlinear springs.

Tools needed: ode45, plot routines.

Description: Sometimes for certain (nonlinear) spring-mass systems, the spring force is not given by Hooke's Law but instead satisfies

$$F_{\text{Spring}} = k\,u + \epsilon\,u^3$$

where $k > 0$ is the spring constant and ϵ is small but may be positive or negative and represents the "strength" of the spring ($\epsilon = 0$ gives Hooke's Law). The spring is called a hardening spring if $\epsilon > 0$ and a softening spring if $\epsilon < 0$.



Questions: Suppose a nonlinear spring-mass system satisfies the initial value problem

$$\begin{cases} u'' + u + \epsilon\,u^3 = 0 \\ u(0) = 0,\ u'(0) = 1 \end{cases} .$$

(1) Let $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ and plot the solutions of the above initial value problem for $0 \le t \le 15$. Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as ϵ increases ? Let $T_1 =$ first time the mass reaches equilibrium after $t = 0$. Estimate T_1 when $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. What appears to happen to T_1 as ϵ increases ?

(2) Let $\epsilon = -0.1, -0.2, -0.3, -0.4$ and plot the solutions of the above initial value problem for $0 \le t \le 15$. Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as ϵ decreases ? Let $T_1 =$ first time the mass reaches equilibrium after $t = 0$. Estimate T_1 when$\epsilon = -0.1, -0.2, -0.3, -0.4$. What appears to happen to T_1 as ϵ decreases ?