

Computer Project # 2

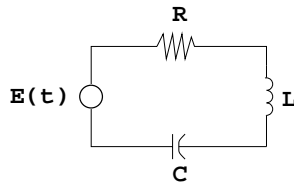
RLC-Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: ode45, plot routines.

Description: If $Q(t)$ = charge on a capacitor at time t in an RLC circuit (with R, L and C being the resistance, inductance and capacitance, respectively, and $E(t)$ = applied voltage), then Kirchoff's Laws give the following 2^{nd} order differential equation for $Q(t)$:

$$(*) \quad LQ''(t) + RQ'(t) + \frac{1}{C}Q(t) = E(t)$$



Questions: Assume $L = 1$, $C = \frac{1}{5}$, $R = 4$ and $E(t) = 8 \cos \omega t$.

- (1) Use **ode45** (and plot routines) to plot the solution of (*) with $Q(0) = 0$ and $Q'(0) = 0$ over the interval $0 \leq t \leq 50$, for these values of ω : $\omega = 0.0, 1.0, 2.0, 4.0, 10.0, 20.0$.
- (2) For each of these 6 plots, find the largest value of $|Q(t)|$ over $30 \leq t \leq 50$ and fill in the table:

ω	Max value of $ Q(t) $ on $30 \leq t \leq 50$
0.0	
1.0	
2.0	
4.0	
10.0	
20.0	

- (3) Does increasing ω appear to increase the maximum charge $|Q(t)|$ on the capacitor ? Interpret this result in terms of an equivalent spring-mass system.

Remark: There is an analogy between spring-mass systems and RLC circuits given by :

SPRING-MASS SYSTEM	RLC CIRCUIT
$mx'' + cx' + kx = F(t)$	$LQ'' + RQ' + \frac{1}{C}Q = E(t)$
x = Displacement	Q = Charge
x' = Velocity	$Q' = I$ = Current
m = Mass	L = Inductance
c = Damping constant	R = Resistance
k = Spring constant	$1/C = (\text{Capacitance})^{-1}$
$F(t)$ = External force	$E(t)$ = Voltage