Use for review

MA 266
FINAL EXAM INSTRUCTIONS
December 16, 2003

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in the instructor's name and the course number.

4. Fill in your NAME and OLD 9-DIGIT PURDUE ID NUMBER, (not your new PUID number) and blacken in the appropriate spaces.

5. Fill in the SECTION NUMBER boxes with the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number, ask your instructor.)

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

11. A table of Laplace Transforms can be found on the last page of the question sheets.

1. If $y(t) = \cos 3t$ is a solution of $y'' - 9y = f(t)$, then $f(t) =$

A. 0

B. $-8\cos 3t$

C. $-10\cos 3t$

D. $-12\cos 3t$

E. $-18\cos 3t$

2. The general solution to $xy' + y = e^{5x}$ is

A. $y = \frac{1}{5}e^{5x} + c$

B. $y = \frac{1}{5x}e^{5x} + cx^{-1}$

C. $y = \frac{1}{6}e^{5x} + ce^{-x}$

D. $y = ce^{5x}$

E. $y = \frac{5}{x}e^{5x} + c$

3. A solution of $\dfrac{dy}{dt} = \dfrac{2y}{t+1}$ with $y(1) = 8$ is

A. $y = (t+1)^2 + 4$

B. $y = 32(t+1)^{-2}$

C. $y = 2\,(t+1)^2$

D. $y = 4\sqrt{2(t+1)}$

E. $y = \sqrt{(t+1)^2 + 60}$

4. An implicit solution of $y' = \dfrac{2x}{y + x^2 y}$ is

A. $y^2 = 2\ln(1+x^2) + C$

B. $y^2 = C\,\ln(1+x^2)$

C. $\frac{1}{2}y^2 = \ln x^2 + C$

D. $y^2 = \ln(1+x^2) + C$

E. $\frac{1}{2}y^2 = \ln|1+x| + C$

5. The substitution $v = y/x$ transforms the equation $\frac{dy}{dx} = \sin(y/x)$ into

A. $v' = \sin(v)$

B. $v' = x\sin(v)$

C. $v' + v = \sin(v)$

D. $xv' + v = \sin(v)$

E. $v' + xv = \sin(v)$

6. Classify the stability of each equilibrium solution for the autonomous differential equation $y' = y(y^2 - 1)$.

A. $y = 0$ stable; $y = 1$ unstable

B. $y = 0$ and $y = 1$ both stable

C. $y = 0$ stable; $y = 1$ unstable; $y = -1$ stable

D. $y = 0$ unstable; $y = 1$ stable

E. $y = 0$ stable; $y = 1$ and $y = -1$ both unstable

7. Solve the initial value problem $2y' - y = e^{-t}$ with $y(0) = a$. For what value(s) of a is the solution bounded on the interval $t > 0$?

A. 2

B. $-\frac{1}{3}$

C. $\frac{1}{3}$

D. all values

E. no values

8. A fish tank contains 20 gallons of a salt solution with a concentration of 5 grams of salt per gallon. A salt solution with a concentration of 10 grams/gallon is added to the tank at a rate of 2 gallons per minute. At the same time, water is drained from the well mixed tank at a rate of 2 gallons per minute. How many grams of salt are in the tank after 10 minutes?

A. $200e^{-1} + 100$

B. $200 - 100e^{-1}$

C. $200e - 100$

D. 100

E. 200

9. An implicit solution of $(y\cos x + 2xe^y) + (\sin x + x^2e^y - 1)y' = 0$, $y(\pi) = 1$ is

A. $y\sin x + x^2e^y = \pi^2 e$

B. $-y\sin x + x^2e^y = \pi^2 e$

C. $-y\sin x + x^2e^y - y = \pi^2 e - 1$

D. $y\sin x + x^2e^y - y = \pi^2 e - 1$

E. $y\sin x + 2xe^y - y = 2\pi e - 1$

10. The largest open interval on which the solution to the initial value problem

$$\begin{cases} (\sin t)\, y' + \dfrac{t}{t-1}\, y = \ln(t-2) \\ y(3) = 0 \end{cases}$$

is guaranteed by the Existence and Uniqueness Theorem to exist is

A. $0 < t < \pi$

B. $1 < t < \pi$

C. $2 < t < \pi$

D. $2 < t < \frac{3\pi}{2}$

E. $2 < t < \infty$

11. If $y(x)$ is the solution of $y'' - y' - 2y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(1) =$

A. $e^{-1} + 2e^2$

B. e^{-1}

C. e^2

D. $e^2 - e^{-1}$

E. $2e^2$

12. The general solution $y(t)$ of the differential equation

$$y'' + 3y' + 2y = 12e^{2t}$$

is

A. $y(t) = c_1 e^{-t} + c_2 e^{-2t} + 2e^{2t}$

B. $y(t) = c_1 e^{-t} + c_2 e^{2t} + e^{-2t}$

C. $y(t) = e^{-t} + e^{2t} + c_1 e^{-2t}$

D. $y(t) = c_1 e^{-t} + c_2 e^{-2t} + e^{2t}$

E. $y(t) = e^{-t} + c_1 e^{-2t} + c_2 e^{2t}$

13. The values of the constant r such that $y = x^r$ solves $x^2y'' + xy' - 2y = 0$ for $x > 0$ are

A. $1 \pm \sqrt{2}$

B. $\pm i\sqrt{2}$

C. $\pm\sqrt{2}$

D. $-1, -2$

E. $1, -2$

14. The proper form of the particular solution of the differential equation

$$y''' + 3y'' + 3y' + y = e^{-t}$$

used in the Method of Undetermined Coefficients is

A. Ae^{-t}

B. $A\cos t + B\sin t$

C. $At\cos t + Bt\sin t$

D. At^2e^{-t}

E. At^3e^{-t}

15. Find the inverse Laplace transform of $\dfrac{e^{-\pi s}}{s^2(s^2+1)}$.

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$
$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

A. $\delta(t-\pi)(t - \sin t)$

B. $\delta(t-\pi)(1 - 4t + 2\sin t)$

C. $u_\pi(t)(t - \frac{1}{2}\cos t + \sin t)$

D. $u_\pi(t)(1 - \frac{1}{2}\cos t + 2\sin t)$

E. $u_\pi(t)(-\pi + t + \sin t)$

16. Use the Laplace Transform to find a function $f(t)$ which satisfies

$$f(t) = 1 - \int_0^t (t-\tau)f(\tau)d\tau.$$

A. $\cos t$

B. e^t

C. $\sin t$

D. e^{-t}

E. 1

17. Find the Laplace transform of

$$f(t) = \begin{cases} 0, & 0 \le t \le 2 \\ t-1, & t > 2 \end{cases}.$$

A. $e^{-2s}(\frac{1}{s^2} - \frac{1}{s})$

B. $e^{-s}(\frac{1}{s^2} - \frac{1}{s})$

C. $e^{s}(\frac{1}{s^2} + \frac{1}{s})$

D. $e^{-2s}(\frac{1}{s^2} + \frac{1}{s})$

E. $e^{2s}(\frac{1}{s^2} + \frac{1}{s})$

18. Find the solution of the differential equation

$$y'' + 4y = \begin{cases} 1, & 0 \le t < 1 \\ -1, & t \ge 1 \end{cases}, \qquad \text{with } y(0) = 0 \text{ and } y'(0) = 0.$$

A. $\frac{1}{4}\cos 2t + \frac{1}{2}u_2(t)\sin 2t$

B. $\frac{1}{2}(1 - \cos 2t) - \frac{1}{2}u_1(t)(1 - \cos 2(t-1))$

C. $1 - \cos 2t - u_1(t)(1 - \cos 2(t-1))$

D. $1 - \cos 2t - u_1(t)(1 - \cos 2(t-1))$

E. $\frac{1}{4}(1 - \cos 2t) - \frac{1}{2}u_1(t)(1 - \cos 2(t-1))$

19. Which of the following differential equations have

$$y(t) = c_1 e^{-t} + c_2 t e^{-t} + c_3 \cos t + c_4 \sin t$$

as a general solution.

A. $y^{(4)} + 2y''' + 2y'' + 2y' + y = 0$
B. $y^{(4)} - 2y'' + y = 0$
C. $y^{(4)} - 2y''' - 2y' + y = 0$
D. $y^{(4)} + 2y''' - 2y' - y = 0$
E. $y^{(4)} - 2y''' + 2y' - y = 0$

20. Find the Laplace Transform $Y(s)$ of the solution to the problem

$$y'' + y = e^{(t-2)} u_2(t) \quad \text{with } y(0) = 1 \text{ and } y'(0) = 0.$$

A. $\dfrac{e^{-2s}}{(s-1)(s^2+1)}$

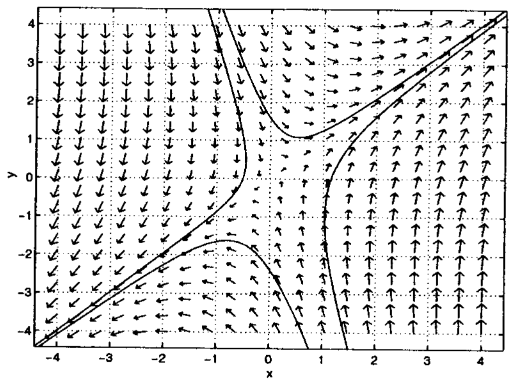
B. $\dfrac{e^{-2s}}{(s-1)(s^2+1)} + \dfrac{1}{s^2+1}$

C. $\dfrac{e^{-2s}}{(s-1)(s^2+1)} + \dfrac{s}{s^2+1}$

D. $\dfrac{e^{-2}e^{-2s}}{(s-1)(s^2+1)} + \dfrac{1}{s^2+1}$

E. $\dfrac{e^{-2}e^{-2s}}{(s-1)(s^2+1)} + \dfrac{s}{s^2+1}$

21. The phase portrait for a linear system of the form $\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}}$, where $\mathbf{A}$ is a 2×2 matrix is as follows.



If r_1 and r_2 denote the eigenvalues of $\mathbf{A}$, then what can you conclude about r_1 and r_2 by examining the phase portrait?

A. r_1 and r_2 are distinct and positive

B. r_1 and r_2 are distinct and negative

C. r_1 and r_2 have opposite signs

D. r_1 and r_2 are complex and have positive real part

E. r_1 and r_2 are equal and negative

22. The function $x_2(t)$ determined by the initial value problem

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 \end{aligned}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $x_2(t) = -\sin t + \cos t$

B. $x_2(t) = \sin t + \cos t$

C. $x_2(t) = \frac{1}{2}(e^t + e^{-t})$

D. $x_2(t) = \cos t$

E. $x_2(t) = ie^{it} - ie^{-it}$

23. The solution of the initial value problem

$$\vec{\mathbf{x}}' = \begin{pmatrix} -2 & 1 \\ -5 & 4 \end{pmatrix} \vec{\mathbf{x}} \qquad \text{with } \vec{\mathbf{x}}(0) = \begin{pmatrix} 1 \\ 3 \end{pmatrix}$$

is

A. $\vec{\mathbf{x}}(t) = \frac{1}{2}\begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t} + \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-t}$

B. $\vec{\mathbf{x}}(t) = \frac{1}{2}\begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{t} + \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$

C. $\vec{\mathbf{x}}(t) = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-3t} + \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{t}$

D. $\vec{\mathbf{x}}(t) = \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{t} + \frac{1}{2}\begin{pmatrix} 1 \\ 1 \end{pmatrix} e^{-3t}$

E. $\vec{\mathbf{x}}(t) = \frac{1}{2}\begin{pmatrix} 1 \\ 5 \end{pmatrix} e^{3t} + \frac{1}{2}\begin{pmatrix} 1 \\ -1 \end{pmatrix} e^{-t}$

24. The 2×2 matrix $\mathbf{A} = \begin{bmatrix} -3 & -1 \\ 2 & -1 \end{bmatrix}$ has complex eigenvalues $r = -2 \pm i$. An eigenvector corresponding to $r = -2 + i$ is $\begin{pmatrix} 1 \\ -1 - i \end{pmatrix}$. The system

$$\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}} + \begin{pmatrix} 3 \\ -4 \end{pmatrix} e^{-2t}$$

has one solution given by $\vec{\mathbf{x}}(t) = \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$. What is the general solution to the system?

A. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{2t}$

B. $c_1 \begin{pmatrix} \cos t \\ \sin t - \cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} \sin t \\ -\sin t - \cos t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

C. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

D. $c_1 \begin{pmatrix} 1 \\ -1 - i \end{pmatrix} e^{(-2+i)t} + c_2 \begin{pmatrix} -1 \\ 1 + i \end{pmatrix} e^{(-2-i)t} + \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

E. $c_1 \begin{pmatrix} \cos t \\ -\cos t \end{pmatrix} e^{-2t} + c_2 \begin{pmatrix} 0 \\ -\sin t \end{pmatrix} e^{-2t} + c_3 \begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-2t}$

25. Find the solution of the initial value problem

$$\vec{\mathbf{x}}' = \mathbf{A}\vec{\mathbf{x}} \quad \text{with } \vec{\mathbf{x}}(0) = \begin{pmatrix} 2 \\ 3 \end{pmatrix},$$

where $\mathbf{A} = \begin{pmatrix} 6 & -4 \\ 1 & 2 \end{pmatrix}$, given that $\vec{\mathbf{a}} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigenvector associated to the repeated eigenvalue 4 for the matrix $\mathbf{A}$, and that $\vec{\mathbf{b}} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ satisfies $(\mathbf{A} - 4\mathbf{I})\vec{\mathbf{b}} = \vec{\mathbf{a}}$.

A. $\begin{pmatrix} 2 \\ 3 \end{pmatrix} e^{4t}$

B. $\begin{pmatrix} 2 - 2t \\ 3 - t \end{pmatrix} e^{4t}$

C. $\begin{pmatrix} 2 + 2t \\ 3 + t \end{pmatrix} e^{4t}$

D. $\begin{pmatrix} 2 - 8t \\ 3 - 4t \end{pmatrix} e^{4t}$

E. $\begin{pmatrix} 2 + 8t \\ 3 + 4t \end{pmatrix} e^{4t}$

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$