SOLUTIONS TO PRACTICE QUESTIONS FOR THE FINAL EXAM

1. $16x^2-4y^8=4\left(4x^2-y^8\right)=4\left((2x)^2-\left(y^4\right)^2\right)=4\left(2x-y^4\right)\left(2x+y^4\right)$

2.

$$\left(\frac{36a^{-4}b^{10}c^2}{a^2c^{-6}}\right)^{-1/2}=\left(36a^{-6}b^{10}c^8\right)^{-1/2}=(36)^{-1/2}\left(a^{-6}\right)^{-1/2}\left(b^{10}\right)^{-1/2}\left(c^8\right)^{-1/2}=$$

$$=\frac{1}{(36)^{1/2}}a^3b^{-5}c^{-4}=\frac{1}{\sqrt{36}}\cdot\frac{a^3}{b^5c^4}=\frac{a^3}{6b^5c^4}$$

3.

$$\frac{3x}{3x+1}-\frac{x}{x-2}=\frac{3x}{(3x+1)}\cdot\frac{(x-2)}{(x-2)}-\frac{x}{(x-2)}\cdot\frac{(3x+1)}{(3x+1)}=$$

$$=\frac{3x(x-2)-x(3x+1)}{(3x+1)(x-2)}=\frac{3x^2-6x-3x^2-x}{(3x+1)(x-2)}=\frac{-7x}{(3x+1)(x-2)}$$

4.

$$(2x+1)^3(2)(3x-5)(3)+(3x-5)^2(3)(2x+1)^2(2)$$
$$=6\left[(2x+1)^3(3x-5)+(3x-5)^2(2x+1)^2\right]$$
$$=6(2x+1)^2\left[(2x+1)(3x-5)+(3x-5)^2\right]$$
$$=6(2x+1)^2(3x-5)\left[(2x+1)+(3x-5)\right]$$
$$=6(2x+1)^2(3x-5)(5x-4)$$
$$=6(3x-5)(5x-4)(2x+1)^2$$

5. $$\frac{xy^{-1}}{(x+y)^{-1}}=\frac{\frac{x}{y}}{\frac{1}{(x+y)}}=\frac{x}{y}\cdot\frac{(x+y)}{1}=\frac{x(x+y)}{y}$$

6.

$$A=P(1+rt)$$
$$A=P+\Pr t$$
$$A-P=\Pr t$$
$$\frac{A-P}{\Pr}=\frac{\Pr t}{\Pr}$$
$$\frac{A-P}{\Pr}=t$$
$$t=\frac{A-P}{\Pr}$$

7.

$$\frac{4}{2p-3}+\frac{10}{4p^2-9}=\frac{1}{2p+3}$$
$$\frac{4}{2p-3}+\frac{10}{(2p-3)(2p+3)}=\frac{1}{2p+3}$$
$$(2p-3)(2p+3)\cdot\left[\frac{4}{2p-3}+\frac{10}{(2p-3)(2p+3)}\right]=\left[\frac{1}{2p+3}\right]\cdot(2p-3)(2p+3)$$
$$(2p+3)\cdot4+10=1\cdot(2p-3)$$
$$4(2p+3)+10=(2p-3)$$
$$8p+12+10=2p-3$$
$$8p-2p=-3-12-10$$
$$6p=-25$$
$$p=-\frac{25}{6}$$

8. $$\frac{\sqrt{x}+5}{\sqrt{x}-5}=\frac{\left(\sqrt{x}+5\right)}{\left(\sqrt{x}-5\right)}\frac{\left(\sqrt{x}+5\right)}{\left(\sqrt{x}+5\right)}=$$

$$=\frac{x+5\sqrt{x}+5\sqrt{x}+25}{x+5\sqrt{x}-5\sqrt{x}-25}=\frac{x+10\sqrt{x}+25}{x-25}$$

9. t = # of hours the other person takes to complete the job.

fraction from 1st person + fraction from 2nd person=whole job

$$\left(\frac{1}{6}\right)\frac{job}{hour}\cdot 4hours+\left(\frac{1}{t}\right)\frac{job}{hour}\cdot 4hours=\left(\frac{1}{4}\right)\frac{job}{hour}\cdot 4hours$$
$$\left(\frac{2}{3}\right)job+\left(\frac{4}{t}\right)job=1job$$
$$\frac{2}{3}+\frac{4}{t}=1$$
$$3t\left(\frac{2}{3}+\frac{4}{t}\right)=1\cdot 3t$$
$$2t+12=3t$$
$$12=t$$
$$t=12$$

10.

$$\begin{cases}y=x+1\\ y^2-x^2=145\end{cases}$$
$$(x+1)^2-x^2=145$$
$$x^2+2x+1-x^2=145$$
$$2x+1=145$$
$$2x=144$$
$$x=72$$

11. let t = # hours truck has been traveling

$$40t=55(t-1)$$
$$40t=55t-55$$
$$55=15t$$

	rate	time	distance
truck	40	t	40t
car	55	t - 1	55(t – 1)

$t=\frac{55}{15}=\frac{11}{3}$ hours, so distance is $40\left(\frac{11}{3}\right)=\frac{440}{3}$ miles

12.

let $x=$ # ml of the 50% solution

let $y=$ total # of ml

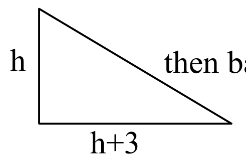
$$\begin{cases}x+40=y\\ x(.50)+40(.20)=y(.25)\end{cases}$$
$$x(.50)+8=(x+40)(.25)$$
$$.50x+8=.25x+10$$
$$.25x=2$$
$$x=8\text{ ml}$$

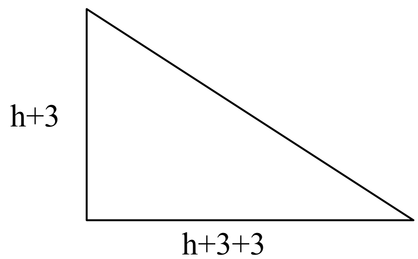
B

13.

Let h = height of original triangle

then base of original triangle = h+3





New:

Area of new triangle $= 14\ in^2$

$\frac{1}{2}(h+3)(h+6) = 14$

$(h+3)(h+6) = 28$

$h^2 + 3h + 6h + 18 = 28$

A

$h^2 + 9h - 10 = 0$

$(h+10)(h-1) = 0$

$h = -10, \ \ h = 1$

Original height = 1 *in*.

Original base = 1 + 3 = 4 *in*.

14.

let t = number of years after 1980 and let V =value
t is the independent variable and V is the dependent variable

points on line $\Rightarrow$ $(1,54)$ and $(3,62)$

slope $\Rightarrow$ $m = \frac{62-54}{3-1} = \frac{8}{2} = 4$

$V - V_1 = m(t - t_1)$

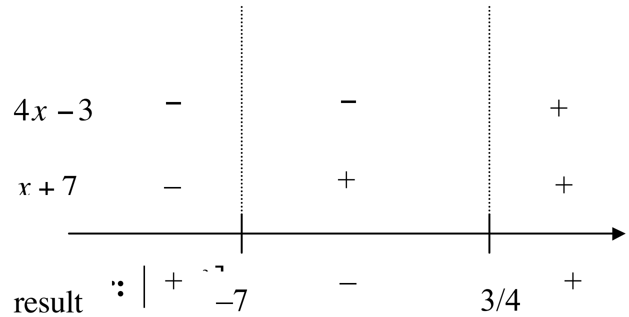
$V - 54 = 4(t-1)$

$V - 54 = 4t - 4$

$V = 4t + 50$

A

15. $(4x-3)(x+7) \le 0$



16.

$|6 - 2x| \le 3$

$-3 \le 6 - 2x \le 3$

$-9 \le -2x \le -3$

$\frac{9}{2} \ge x \ge \frac{3}{2}$

C

$\frac{3}{2} \le x \le \frac{9}{2}$

17.

$A\ (1,-2)$, Midpoint $M\ (2,3)$, $B\ (x,y)$

Midpoint $\Rightarrow \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

Midpoint of $\overline{AB} \Rightarrow \left(\frac{1+x}{2}, \frac{-2+y}{2}\right) \Rightarrow (2,3)$

$\frac{1+x}{2} = 2, \qquad \frac{-2+y}{2} = 3$

$1 + x = 4, \qquad -2 + y = 6$

$x = 3, \qquad y = 8$

so $B\ (3,8)$

C

18.

slope of line $\Rightarrow$ $m = -\frac{1}{3}$

slope of line perpendicular $\Rightarrow$ $m = 3$

D

19.

$2x - 3y = 7$

$-3y = -2x + 7$

$y = \frac{2}{3}x - \frac{7}{3}$

slope $m = \frac{2}{3}$

slope of parallel line $m = \frac{2}{3}$

point is $(2, -1)$; $m = \frac{2}{3}$

$y = mx + b$

$-1 = \left(\frac{2}{3}\right)(2) + b$

C

$-1 = \frac{4}{3} + b$

$b = -\frac{7}{3}$ so $y = \frac{2}{3}x - \frac{7}{3}$

20.

Center $\Rightarrow$ $(0,2)$ $\quad (x-h)^2 + (y-k)^2 = r^2$

radius $= 2$ $\quad (x-0)^2 + (y-2)^2 = 2^2$

$x^2 + (y-2)^2 = 4$

B

$x^2 + y^2 - 4y + 4 = 4$

$x^2 + y^2 - 4y = 0$

21.

$f(x) = 1 - \sqrt{x}$, $\quad g(x) = \frac{1}{x}$

D

$(g \circ f)(x) = g[f(x)] = g\left(1 - \sqrt{x}\right) = \frac{1}{1 - \sqrt{x}}$

22.

$f(x) = \frac{x}{x^2 + 1}$

D

$\frac{1}{f(3)} = \frac{1}{\frac{3}{(3)^2 + 1}} = \frac{1}{\frac{3}{10}} = \frac{10}{3}$

23.

$y = \frac{1}{3x - 2}$

$x = \frac{1}{3y - 2}$

$x(3y - 2) = 1$

$3xy - 2x = 1$

$3xy = 1 + 2x$

$y = \frac{1 + 2x}{3x} = f^{-1}(x)$

24.

$f(x) = x^2 - 2x + 4$

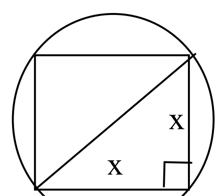
$\frac{f(x+h) - f(x)}{h} = \frac{(x+h)^2 - 2(x+h) + 4 - (x^2 - 2x + 4)}{h}$

A

$= \frac{x^2 + 2xh + h^2 - 2x - 2h + 4 - x^2 + 2x - 4}{h} = \frac{2xh + h^2 - 2h}{h}$

$= \frac{h(2x + h - 2)}{h} = 2x + h - 2$

25.



Let A = area of circle

Area of circle $\Rightarrow$ $A(r) = \pi r^2$

Diameter (d) of circle $\Rightarrow$ $x^2 + x^2 = d^2$

$2x^2 = d^2$

A

$d = \pm\sqrt{2x^2}$

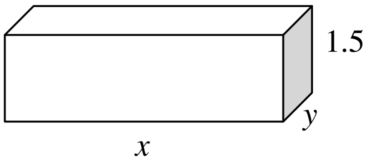
$d = x\sqrt{2}$

Radius (r) of circle $\Rightarrow$ $r = \frac{x\sqrt{2}}{2}$

So, $A(x) = \pi\left(\frac{x\sqrt{2}}{2}\right)^2 = \pi\left(\frac{x^2(2)}{4}\right)$

$= \frac{\pi x^2}{2}$ or $\frac{\pi}{2}x^2$

26.



Volume $= 6 ft.^3$

$xy(1.5) = 6$

$y = \dfrac{6}{1.5x}$ **B**

$y = \dfrac{4}{x}$

27.

$T = k\dfrac{a^3}{\sqrt{d}}$

$4 = k\dfrac{2^3}{\sqrt{9}}$

$4 = k\dfrac{8}{3}$

$k = \dfrac{4}{1}\cdot\dfrac{3}{8}$

$k = \dfrac{3}{2}$ →

$T = \dfrac{3}{2}\cdot\dfrac{(-1)^3}{\sqrt{4}}$

$T = \dfrac{3}{2}\cdot\left(-\dfrac{1}{2}\right)$ **A**

$T = -\dfrac{3}{4}$

28.

$x^2 - 4x - 2y - 4 = 0$

$2y = x^2 - 4x - 4$

$2y = (x^2 - 4x + 4) - 4 - 4$

$2y = (x-2)^2 - 8$

$y = \frac{1}{2}(x-2)^2 - 4$

$y = a(x-h)^2 + k$

$Vertex(h,k) = (2,-4)$

29.

Vertex $\Rightarrow V(0,2)$

point on parabola $\Rightarrow (1,0)$

$y = a(x-h)^2 + k$

$y = a(x-0)^2 + 2$

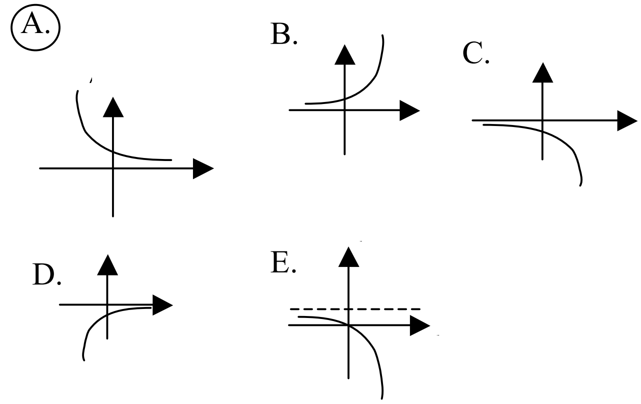
$y = ax^2 + 2$ **B**

$0 = a(1)^2 + 2$

$a = -2$

$y = -2x^2 + 2$

30.



31.

$\log_b y^3 + \log_b y^2 - \log_b y^4 = \log_b(y^3y^2) - \log_b y^4$

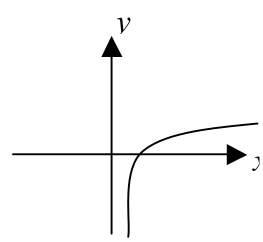
$= \log_b y^5 - \log_b y^4 = \log_b\left(\dfrac{y^5}{y^4}\right) = \log_b y$ **B**

32.

$f(x) = \log_a x$ if $a > 1$

example: if $a = 2$, then $f(x) = \log_2 x$, **D**

Graph of $y = \log_2 x \Rightarrow 2^y = x$



f is increasing, f does not have a as an x - intercept (the x - int. is (1,0)), f does not have a y - intercept, the domain of f is $(0,\infty)$.

33.

$\log\left(\dfrac{432}{(\sqrt{.095})(\sqrt[3]{72.1})}\right) = \log\left(\dfrac{432}{(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}}\right)$

$= \log 432 - \left(\log\left[(.095)^{\frac{1}{2}}(72.1)^{\frac{1}{3}}\right]\right)$ **B**

$= \log 432 - \left(\log(.095)^{\frac{1}{2}} + \log(72.1)^{\frac{1}{3}}\right)$

$= \log 432 - \dfrac{1}{2}\log .095 - \dfrac{1}{3}\log 72.1$