MA 165 FINAL EXAM PRACTICE QUESTIONS

1. $\lim_{x\to 1} \dfrac{x^2-1}{x^2-x} =$ A. -1 B. 0 C. 1 D. 2 E. Does not exist

2. If $y = (x^2+1)\tan x$, then $\dfrac{dy}{dx} =$ A. $2x\tan x + (x^2+1)\sec^2 x$ B. $2x\sec^2 x$ C. $2x\tan x + (x^2+1)\tan x$ D. $2x\tan x + 2x\sec^2 x$ E. $2x\tan x$

3. If $h(x) = \begin{cases} x^2+a, & \text{for } x < -1 \\ x^3-8 & \text{for } x \geq -1 \end{cases}$ determine all values of a so that h is continuous for all values of x. A. $a=-1$ B. $a=-8$ C. $a=-9$ D. $a=-10$ E. There are no values of a.

4. Evaluate $\lim_{x\to 0^+} x\cos\left(\dfrac{1}{x}\right)$. (Hint: $-1 \leq \cos\left(\dfrac{1}{x}\right) \leq 1$ for all $x \neq 0$.) A. 0 B. 1 C. -1 D. $\dfrac{\pi}{2}$ E. Does not exist

5. If $f(x) = \dfrac{1}{x+3}$, then $\lim_{x\to 1} \dfrac{f(x)-f(1)}{x-1} =$ A. $\dfrac{1}{4}$ B. $\dfrac{1}{16}$ C. $-\dfrac{1}{16}$ D. $-\dfrac{1}{4}$ E. Does not exist

6. The equation $x^3-x-5=0$ has one root in the interval $(-2,2)$. This root is in the interval: A. $(-2,-1)$ B. $(-1,0)$ C. $(0,1)$ D. $(1,2)$ E. $(-1,1)$

7. If $f(x) = \dfrac{1-x}{1+x}$, then $f'(1) =$ A. -1 B. $-\dfrac{1}{2}$ C. 0 D. $\dfrac{1}{2}$ E. 1

8. If $y = \ln(1-x^2) + \sin^2 x$, then $\dfrac{dy}{dx} =$ A. $\dfrac{1}{1-x^2} + \cos^2 x$ B. $\dfrac{1}{1-x^2} + 2\sin x\cos x$ C. $\dfrac{1}{1-x^2} + 2\sin x$ D. $\dfrac{-2x}{1-x^2} + \cos^2 x$ E. $\dfrac{-2x}{1-x^2} + 2\sin x\cos x$

9. Find $f''(x)$ if $f(x) = \dfrac{1-x}{1+x}$ A. $\dfrac{4}{(1+x)^3}$ B. $\dfrac{-4}{(1+x)^3}$ C. $-\dfrac{4x}{(1+x)^3} + \dfrac{2}{(1+x)^2}$ D. $\dfrac{2(1+x)^2 - 2x(1+x)}{(1+x)^4}$ E. -1

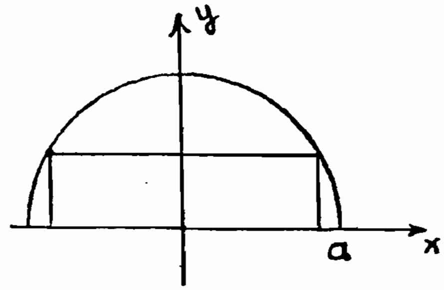
10. Assume that y is defined implicitly as a differentiable function of x by the equation $xy^2 - x^2 + y + 5 = 0$. Find $\dfrac{dy}{dx}$ at $(-2,1)$. A. 9 B. $\dfrac{-5}{3}$ C. 1 D. 2 E. $\dfrac{5}{3}$

11. Find the maximum and minimum values of the function $f(x) = 3x^2+6x-10$ on the interval $-2 \leq x \leq 2$. A. max is 14, min is -10. B. max is -10, min is -13 C. max is 14, min is -13 D. no max., min is -10 E. max is 14, no min.

12. For a differentiable function $f(x)$ it is known that $f(3) = 5$ and $f'(3) = -2$. Use a linear approximation to get the approximate value of $f(3.02)$. A. 6.02 B. 5.02 C. 5.04 D. 3 E. 4.96.

13. Water is withdrawn from a conical reservoir, 8 feet in diameter and 10 feet deep (vertex down) at the constant rate of 5 ft^3/min. How fast is the water level falling when the depth of the water in the reservoir is 5 ft? ($V = \dfrac{1}{3}\pi r^2 h$). A. $\dfrac{15}{16\pi}$ ft/min B. $\sqrt{\dfrac{3}{\pi}}$ ft/min C. $\dfrac{2}{\pi}$ ft/min D. $5\sqrt[3]{3/4\pi}$ ft/min E. $\dfrac{5}{4\pi}$ ft/min.

14. A rectangle is inscribed in the upper half of the circle $x^2 + y^2 = a^2$ as shown at right. Calculate the area of the largest such rectangle.
A. $\frac{a^2}{2}$ B. $3a\sqrt{2}$ C. $2a^2$ D. $4a^2$ E. a^2.



15. Given that $f(x)$ is differentiable for all x, $f(2) = 4$, and $f(7) = 10$, then the Mean Value Theorem states that there is a number c such that A. $2 < c < 7$ and $f'(c) = \frac{6}{5}$ B. $2 < c < 7$ and $f'(c) = \frac{5}{6}$ C. $4 < c < 10$ and $f'(c) = \frac{6}{5}$ D. $2 < c < 7$ and $f'(c) = 0$ E. $4 < c < 10$ and $f'(c) = 0$.

16. Suppose that the mass of a radioactive substance decays from 18 gms to 2 gms in 2 days. How long will it take for 12 gms of this substance to decay to 4 gms?
A. $\frac{\ln 3}{\ln 2}$ days B. 1 day C. $\frac{\ln 2}{\ln 3}$ days D. 2 days E. $(\ln 3)^2$ days

17. Which of the following is/are true about the function $g(x) = 4x^3 - 3x^4$? (1) g is decreasing for $x > 1$. (2) g has a relative extreme value at $(0, 0)$. (3) the graph of g is concave up for all $x < 0$. A. (1), (2) and (3) B. only (2) C. only (1) D. (1) and (2) E. (1) and (3).

18. Find where the function $f(x) = 2/\sqrt{1 + x^2}$ is increasing A. all x B. no x C. $x < 0$ D. $x > 0$ E. $x = 0$.

19. Let f be a function whose <u>derivative</u>, f', is given by $f'(x) = (x-1)^2(x+2)(x-5)$. The function has A. a relative maximum at $x = -2$ and a relative minimum at $x = 5$. B. a relative maximum at $x = 5$ and a relative minimum at $x = -2$. C. relative maxima at $x = 1$, $x = -2$ and a relative minimum at $x = 5$. D. a relative maximum at $x = 5$ and relative minima at $x = 1$, $x = -2$ E. a relative maximum at $x = 1$ and relative minima at $x = -2$, $x = 5$.

20. Find $\frac{d}{dx}\int_1^{2x} \sqrt{t^2 + 1}\,dt$ at $x = \sqrt{2}$. A. 6 B. 3 C. $\sqrt{2}$ D. $\sqrt{4x^2 + 1}$ E. $\frac{1}{2\sqrt{3}}$.

21. $\int_3^4 x\sqrt{25 - x^2}\,dx =$ A. 0 B. -37 C. $\frac{37}{3}$ D. $-\frac{74}{3}$ E. $\frac{7}{12}$

22. $\lim_{x\to\infty} \frac{x^2 + 2x}{3x^2 + 4} =$ A. 1 $\frac{3}{7}$ C. $\frac{1}{4}$ D. 0 E. $\frac{1}{3}$.

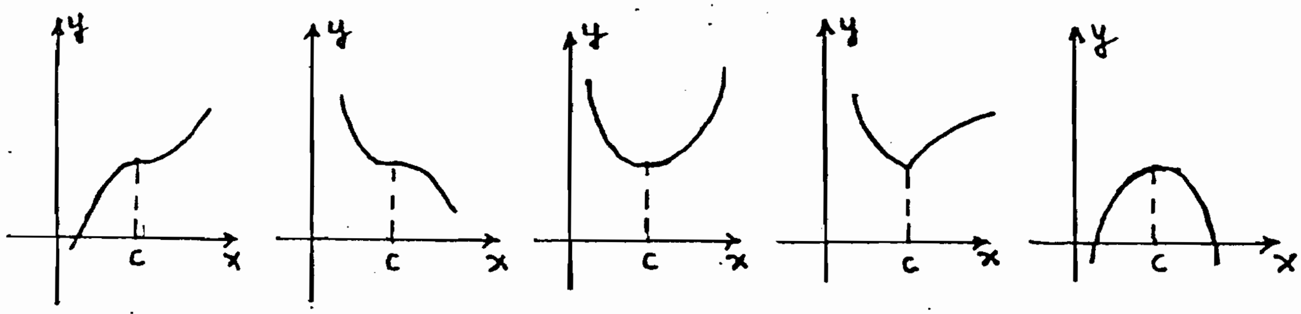
23. $\lim_{x\to 0} \frac{2x - \sin^{-1} x}{2x + \tan^{-1} x} =$ A. $\frac{1}{2}$ B. 2 C. $\frac{1}{3}$ D. 1 E. 0

24. Suppose that a function f has the following properties:

$$f''(x) > 0 \text{ for } x < c$$
$$f'(c) = 0$$
$$\text{and } f'(x) < 0 \text{ for } x > c.$$

Which of the following could be the graph of f?

A. B. C. D. E.



25. Let R be the region between the graph of $y = \frac{1}{x}$ and the x-axis, from $x = a$ to $x = b$ $(0 < a < b)$. If the vertical line $x = c$ cuts R into two parts of equal area, then $c =$
A. $\sqrt{ab}$ B. $\frac{a+b}{2}$ C. $\frac{\ln a + \ln b}{2}$ D. $\ln\left(\frac{a+b}{2}\right)$ E. $\ln\left(\frac{b-a}{2}\right)$

26. The area of the region between the graph of $y = \frac{1}{1+x^2}$ and the x-axis, from $x = -\sqrt{3}$ to $x = 1$ is A. $\frac{\pi}{2}$ B. $\frac{3\pi}{4}$ C. $\frac{5\pi}{12}$ D. $\frac{\pi}{3}$ E. $\frac{7\pi}{12}$

27. $\frac{d}{dx}(e^{2x} \ln\sqrt{1+x}) =$ A. $e^{2x}\ln(1+x) + \frac{e^{2x}}{2(1+x)}$ B. $\frac{e^{2x}}{\sqrt{1+x}} + 2e^{2x}\ln\sqrt{1+x}$
C. $\frac{1}{2}e^{2x}\ln(1+x) + \frac{e^{2x}}{2(1+x)}$ D. $\frac{2e^{2x}}{\sqrt{1+x}}$ E. $\frac{e^{2x}}{1+x}$

28. $\frac{d}{dx}x^{\sin x} =$ A. $(\cos x)x^{\sin x}$ B. $(\sin x)x^{\sin x - 1}$ C. $x^{\cos x}$ D. $x^{\sin x}[\frac{\sin x}{x} + (\cos x)\ln x]$
E. $(\ln x)x^{\sin x}$

29. $\frac{d}{dx}\tan^{-1} e^{3x} =$ A. $\frac{1}{1+e^{3x}}$ B. $\frac{e^{3x}}{1+e^{3x}}$ C. $\frac{3e^{3x}}{1+e^{6x}}$ D. $\frac{3e^{3x}}{1+e^{9x^2}}$ E. $\frac{3e^{3x}}{\sqrt{1-e^{6x}}}$

30. $\int_0^{\sqrt{3}} \frac{1}{\sqrt{4-x^2}}dx =$ A. $\frac{\pi}{2}$ B. $\frac{\pi}{6}$ C. $\sin^{-1}\sqrt{3}$ D. $\frac{\pi}{3}$ E. 1

31. $\int_0^4 \frac{x}{\sqrt{1+2x}}dx =$ A. $\frac{7}{2}$ B. $\frac{10}{3}$ C. $\frac{11}{4}\tan^{-1} 3$ D. 3 E. 4

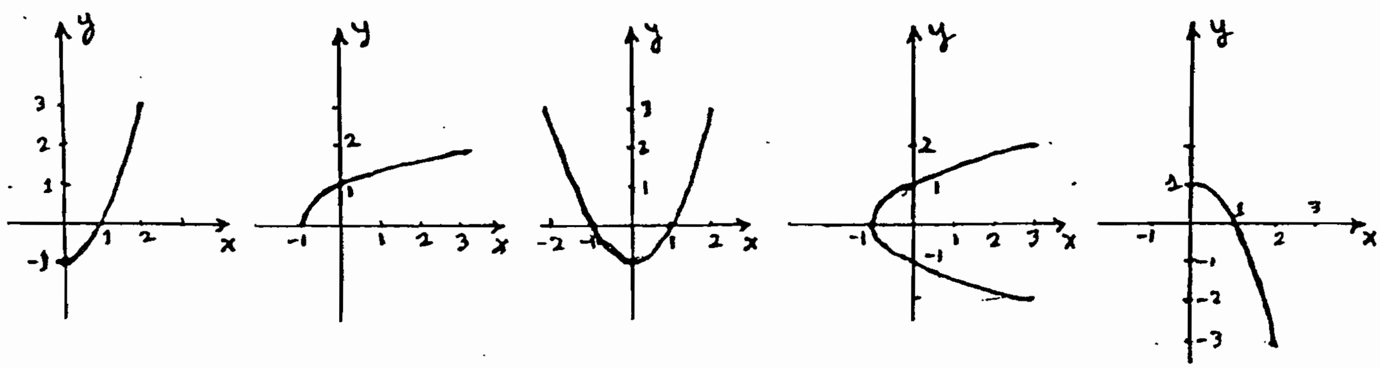
32. $\int_0^1 \frac{e^x}{1+e^x}dx =$ A. $\ln\frac{1+e}{2}$ B. $\ln(1+e)$ C. $\frac{1}{2}$ D. $1 - \ln 2$ E. e

33. An equation of the parabola with horizontal axis, vertex $(-2, 3)$, and containing the point $(1, 2)$ is
A. $(y-3)^2 = \frac{1}{12}(x+2)$ B. $(y+2)^2 = -8(x-3)$ C. $(y-3)^2 = \frac{1}{3}(x+2)$
D. $(x+2)^2 = -9(y-3)$ E. $(x+2)^2 = -\frac{9}{4}(y-3)$

34. The ellipse $16(x-3)^2 + 25(y-7)^2 = 400$ has one focus at A. $(6, 7)$ B. $(7, 7)$
C. $(3, 10)$ D. $(3, 11)$ E. $(3, 12)$

35. The asymptotes of the hyperbola $9x^2 - 4y^2 - 36x - 8y - 4 = 0$ have equations:
A. $2x - 3y - 7 = 0, 2x + 3y - 1 = 0$ B. $3x - 2y - 8 = 0, 3x + 2y - 4 = 0$
C. $3x - 2y = 0, 3x + 2y = 0$ D. $3x - 2y + 7 = 0, 3x + 2y - 1 = 0$
E. $2x - 3y + 8 = 0, 2x + 3y - 4 = 0$

36. If $f(x) = x^2 - 1$, $0 \le x \le 2$, then the graph of $y = f^{-1}(x)$ is
A. B. C. D. E.



Answers: 1.D, 2.A, 3.D, 4.A, 5.C, 6.D, 7.B, 8.E, 9.A, 10.E, 11.C, 12.E, 13.E, 14.E, 15.A, 16.B, 17.C, 18.C, 19.A, 20.A, 21.C, 22.E, 23.C, 24.B, 25.A, 26.E, 27.A, 28.D, 29.C, 30.D, 31.B, 32.A, 33.C, 34.A, 35.B, 36.B