

FINAL EXAM FALL 2006

MA 261 PRACTICE PROBLEMS

ANSWERS AT END

1. If the line ℓ has symmetric equations

$$\frac{x-1}{2} = \frac{y}{-3} = \frac{z+2}{7},$$

find a vector equation for the line ℓ' that contains the point $(2, 1, -3)$ and is parallel to ℓ .

- A. $\vec{r} = (1 + 2t)\vec{i} - 3t\vec{j} + (-2 + 7t)\vec{k}$ B. $\vec{r} = (2 + t)\vec{i} - 3\vec{j} + (7 - 2t)\vec{k}$
 C. $\vec{r} = (2 + 2t)\vec{i} + (1 - 3t)\vec{j} + (-3 + 7t)\vec{k}$ D. $\vec{r} = (2 + 2t)\vec{i} + (-3 + t)\vec{j} + (7 - 3t)\vec{k}$
 E. $\vec{r} = (2 + t)\vec{i} + \vec{j} + (7 - 3t)\vec{k}$

2. Find parametric equations of the line containing the points $(1, -1, 0)$ and $(-2, 3, 5)$.

- A. $x = 1 - 3t, y = -1 + 4t, z = 5t$ B. $x = t, y = -t, z = 0$
 C. $x = 1 - 2t, y = -1 + 3t, z = 5t$ D. $x = -2t, y = 3t, z = 5t$
 E. $x = -1 + t, y = 2 - t, z = 5$

3. Find an equation of the plane that contains the point $(1, -1, -1)$ and has normal vector $\frac{1}{2}\vec{i} + 2\vec{j} + 3\vec{k}$.

- A. $x - y - z + \frac{9}{2} = 0$ B. $x + 4y + 6z + 9 = 0$ C. $\frac{x-1}{\frac{1}{2}} = \frac{y+1}{2} = \frac{z+1}{3}$
 D. $x - y - z = 0$ E. $\frac{1}{2}x + 2y + 3z = 1$

4. Find an equation of the plane that contains the points $(1, 0, -1)$, $(-5, 3, 2)$, and $(2, -1, 4)$.

- A. $6x - 11y + z = 5$ B. $6x + 11y + z = 5$ C. $11x - 6y + z = 0$
 D. $\vec{r} = 18\vec{i} - 33\vec{j} + 3\vec{k}$ E. $x - 6y - 11z = 12$

5. Find parametric equations of the line tangent to the curve $\vec{r}(t) = t\vec{i} + t^2\vec{j} + t^3\vec{k}$ at the point $(2, 4, 8)$

- A. $x = 2 + t, y = 4 + 4t, z = 8 + 12t$ B. $x = 1 + 2t, y = 4 + 4t, z = 12 + 8t$
 C. $x = 2t, y = 4t, z = 8t$ D. $x = t, y = 4t, z = 12t$ E. $x = 2 + t, y = 4 + 2t, z = 8 + 3t$

6. The position function of an object is

$$\vec{r}(t) = \cos t\vec{i} + 3 \sin t\vec{j} - t^2\vec{k}$$

Find the velocity, acceleration, and speed of the object when $t = \pi$.

- | | Velocity | Acceleration | Speed |
|----|------------------------------------|---------------------------|----------------------|
| A. | $-\vec{i} - \pi^2\vec{k}$ | $-3\vec{j} - 2\pi\vec{k}$ | $\sqrt{1 + \pi^4}$ |
| B. | $\vec{i} - 3\vec{j} + 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{10 + 4\pi^2}$ |
| C. | $3\vec{j} - 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{9 + 4\pi^2}$ |
| D. | $-3\vec{j} - 2\pi\vec{k}$ | $-\vec{i} - 2\vec{k}$ | $\sqrt{9 + 4\pi^2}$ |
| E. | $\vec{i} - 2\vec{k}$ | $-3\vec{j} - 2\pi\vec{k}$ | $\sqrt{5}$ |

7. A smooth parametrization of the semicircle which passes through the points $(1, 0, 5)$, $(0, 1, 5)$ and $(-1, 0, 5)$ is
- A. $\vec{r}(t) = \sin t \vec{i} + \cos t \vec{j} + 5\vec{k}, 0 \leq t \leq \pi$ B. $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} + 5\vec{k}, 0 \leq t \leq \pi$
 C. $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} + 5\vec{k}, \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$ D. $\vec{r}(t) = \cos t \vec{i} + \sin t \vec{j} + 5\vec{k}, 0 \leq t \leq \frac{\pi}{2}$
 E. $\vec{r}(t) = \sin t + \cos t \vec{j} + 5\vec{k}, \frac{\pi}{2} \leq t \leq \frac{3\pi}{2}$
8. The length of the curve $\vec{r}(t) = \frac{2}{3}(1+t)^{\frac{3}{2}}\vec{i} + \frac{2}{3}(1-t)^{\frac{3}{2}}\vec{j} + t\vec{k}, -1 \leq t \leq 1$ is
- A. $\sqrt{3}$ B. $\sqrt{2}$ C. $\frac{1}{2}\sqrt{3}$ D. $2\sqrt{3}$ E. $\sqrt{2}$
9. The level curves of the function $f(x, y) = \sqrt{1 - x^2 - 2y^2}$ are
- A. circles B. lines C. parabolas D. hyperbolas E. ellipses
10. The level surface of the function $f(x, y, z) = z - x^2 - y^2$ that passes through the point $(1, 2, -3)$ intersects the (x, z) -plane ($y = 0$) along the curve
- A. $z = x^2 + 8$ B. $z = x^2 - 8$ C. $z = x^2 + 5$ D. $z = -x^2 - 8$
 E. does not intersect the (x, z) -plane
11. Match the graphs of the equations with their names:
- (1) $x^2 + y^2 + z^2 = 4$ (a) paraboloid
 (2) $x^2 + z^2 = 4$ (b) sphere
 (3) $x^2 + y^2 = z^2$ (c) cylinder
 (4) $x^2 + y^2 = z$ (d) double cone
 (5) $x^2 + 2y^2 + 3z^2 = 1$ (e) ellipsoid
- A. 1b, 2c, 3d, 4a, 5e B. 1b, 2c, 3a, 4d, 5e C. 1e, 2c, 3d, 4a, 5b
 D. 1b, 2d, 3a, 4c, 5e E. 1d, 2a, 3b, 4e, 5c
12. Suppose that $w = u^2/v$ where $u = g_1(t)$ and $v = g_2(t)$ are differentiable functions of t . If $g_1(1) = 3$, $g_2(1) = 2$, $g_1'(1) = 5$ and $g_2'(1) = -4$, find $\frac{dw}{dt}$ when $t = 1$.
- A. 6 B. $33/2$ C. -24 D. 33 E. 24
13. If $w = e^{uv}$ and $u = r + s$, $v = rs$, find $\frac{\partial w}{\partial r}$.
- A. $e^{(r+s)rs}(2rs + r^2)$ B. $e^{(r+s)rs}(2rs + s^2)$ C. $e^{(r+s)rs}(2rs + r^2)$
 D. $e^{(r+s)rs}(1 + s)$ E. $e^{(r+s)rs}(r + s^2)$.

23. The differential df of the function $f(x, y, z) = xe^{y^2 - z^2}$ is
- $df = xe^{y^2 - z^2} dx + xe^{y^2 - z^2} dy + xe^{y^2 - z^2} dz$
 - $df = xe^{y^2 - z^2} dx dy dz$
 - $df = e^{y^2 - z^2} dx - 2xye^{y^2 - z^2} dy + 2xze^{y^2 - z^2} dz$
 - $df = e^{y^2 - z^2} dx + 2xye^{y^2 - z^2} dy - 2xze^{y^2 - z^2} dz$
 - $df = e^{y^2 - z^2}(1 + 2xy - 2xz)$
24. The function $f(x, y) = 2x^3 - 6xy - 3y^2$ has
- a local minimum and a saddle point
 - a local maximum and a saddle point
 - a local minimum and a local maximum
 - two saddle points
 - two local minima.
25. Consider the problem of finding the minimum value of the function $f(x, y) = 4x^2 + y^2$ on the curve $xy = 1$. In using the method of Lagrange multipliers, the positive value of λ (even though it is not needed) will be
- 2
 - 2
 - $\sqrt{2}$
 - $\frac{1}{\sqrt{2}}$
 - 4.
26. Evaluate the iterated integral $\int_1^3 \int_0^x \frac{1}{x} dy dx$.
- $-\frac{8}{9}$
 - 2
 - $\ln 3$
 - 0
 - $\ln 2$.
27. Consider the double integral, $\iint_R f(x, y) dA$, where R is the portion of the disk $x^2 + y^2 \leq 1$, in the upper half-plane, $y \geq 0$. Express the integral as an iterated integral.
- $\int_{-1}^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x, y) dy dx$
 - $\int_{-1}^0 \int_0^{\sqrt{1-x^2}} f(x, y) dy dx$
 - $\int_{-1}^1 \int_0^{\sqrt{1-x^2}} f(x, y) dy dx$
 - $\int_0^1 \int_{-\sqrt{1-x^2}}^{\sqrt{1-x^2}} f(x, y) dy dx$
 - $\int_0^1 \int_0^{\sqrt{1-x^2}} f(x, y) dy dx$.
28. Find a and b for the correct interchange of order of integration:
- $$\int_0^2 \int_{x^2}^{2x} f(x, y) dy dx = \int_0^4 \int_a^b f(x, y) dx dy.$$
- $a = y^2, b = 2y$
 - $a = \frac{y}{2}, b = \sqrt{y}$
 - $a = \frac{y}{2}, b = y$
 - $a = \sqrt{y}, b = \frac{y}{2}$
 - cannot be done without explicit knowledge of $f(x, y)$.
29. Evaluate the double integral $\iint_R y dA$, where R is the region of the (x, y) -plane inside the triangle with vertices $(0, 0)$, $(2, 0)$ and $(2, 1)$.
- 2
 - $\frac{8}{3}$
 - $\frac{2}{3}$
 - 1
 - $\frac{1}{3}$.
30. The volume of the solid region in the first octant bounded above by the parabolic sheet $z = 1 - x^2$, below by the xy plane, and on the sides by the planes $y = 0$ and $y = x$ is given by the double integral
- $\int_0^1 \int_0^x (1 - x^2) dy dx$
 - $\int_0^1 \int_0^{1-x^2} x dy dx$
 - $\int_{-1}^1 \int_{-x}^x (1 - x^2) dy dx$
 - $\int_0^1 \int_x^0 (1 - x^2) dy dx$
 - $\int_0^1 \int_x^{1-x^2} dy dx$.

31. The area of one leaf of the three-leaved rose bounded by the graph of $r = 5 \sin 3\theta$ is
- A. $\frac{5\pi}{6}$ B. $\frac{25\pi}{12}$ C. $\frac{25\pi}{6}$ D. $\frac{5\pi}{3}$ E. $\frac{25\pi}{3}$.
32. Find the area of the portion of the plane $x + 3y + 2z = 6$ that lies in the first octant.
- A. $3\sqrt{11}$ B. $6\sqrt{7}$ C. $6\sqrt{14}$ D. $3\sqrt{14}$ E. $6\sqrt{11}$.
33. A solid region in the first octant is bounded by the surfaces $z = y^2$, $y = x$, $y = 0$, $z = 0$ and $x = 4$. The volume of the region is
- A. 64 B. $\frac{64}{3}$ C. $\frac{32}{3}$ D. 32 E. $\frac{16}{3}$.
34. An object occupies the region bounded above by the sphere $x^2 + y^2 + z^2 = 32$ and below by the cone $z = \sqrt{x^2 + y^2}$. The mass density at any point of the object is equal to its distance from the xy plane. Set up a triple integral in rectangular coordinates for the total mass m of the object.
- A. $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$ B. $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$
- C. $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{-\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$ D. $\int_0^4 \int_0^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} z \, dz \, dy \, dx$
- E. $\int_{-4}^4 \int_{-\sqrt{16-x^2}}^{\sqrt{16-x^2}} \int_{\sqrt{x^2+y^2}}^{\sqrt{32-x^2-y^2}} xy \, dz \, dy \, dx$.
35. Do problem 34 in spherical coordinates.
- A. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \cos \varphi \sin \varphi \, d\rho \, d\varphi \, d\theta$ B. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos \varphi \sin \varphi \, d\rho \, d\varphi \, d\theta$
- C. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho^3 \sin^2 \varphi \, d\rho \, d\varphi \, d\theta$ D. $\int_0^{2\pi} \int_0^{\frac{\pi}{2}} \int_0^{\sqrt{32}} \rho^3 \cos \varphi \sin \varphi \, d\rho \, d\varphi \, d\theta$
- E. $\int_0^{2\pi} \int_0^{\frac{\pi}{4}} \int_0^{\sqrt{32}} \rho \cos \varphi \, d\rho \, d\varphi \, d\theta$.
36. The double integral $\int_0^1 \int_0^{\sqrt{1-x^2}} y^2(x^2 + y^2)^3 \, dy \, dx$ when converted to polar coordinates becomes
- A. $\int_0^\pi \int_0^1 r^9 \sin^2 \theta \, dr \, d\theta$ B. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin^2 \theta \, dr \, d\theta$ C. $\int_0^\pi \int_0^1 r^8 \sin \theta \, dr \, d\theta$
- D. $\int_0^{\frac{\pi}{2}} \int_0^1 r^8 \sin \theta \, dr \, d\theta$ E. $\int_0^{\frac{\pi}{2}} \int_0^1 r^9 \sin^2 \theta \, dr \, d\theta$.
37. Which of the triple integrals converts $\int_{-2}^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} \int_{\sqrt{x^2+y^2}}^2 dz \, dy \, dx$ from rectangular to cylindrical coordinates?
- A. $\int_0^\pi \int_0^2 \int_r^2 r \, dz \, dr \, d\theta$ B. $\int_0^{2\pi} \int_0^2 \int_r^2 r \, dz \, dr \, d\theta$ C. $\int_0^{2\pi} \int_{-2}^2 \int_r^2 r \, dz \, dr \, d\theta$
- D. $\int_0^\pi \int_0^2 \int_r^2 r \, dz \, dr \, d\theta$ E. $\int_0^{\frac{2\pi}{2}} \int_{-2}^2 \int_r^2 r \, dz \, dr \, d\theta$.
38. If D is the solid region above the xy -plane that is between $z = \sqrt{4-x^2-y^2}$ and $z = \sqrt{1-x^2-y^2}$, then $\iiint_D \sqrt{x^2+y^2+z^2} \, dV =$
- A. $\frac{14\pi}{3}$ B. $\frac{16\pi}{3}$ C. $\frac{15\pi}{2}$ D. 8π E. 15π .

39. Determine which of the vector fields below are conservative, i. e. $\vec{F} = \text{grad } f$ for some function f .

1. $\vec{F}(x, y) = (xy^2 + x)\vec{i} + (x^2y - y^2)\vec{j}$.
2. $\vec{F}(x, y) = \frac{x}{y}\vec{i} + \frac{y}{x}\vec{j}$.
3. $\vec{F}(x, y, z) = ye^z\vec{i} + (xe^z + e^y)\vec{j} + (xy + 1)e^z\vec{k}$.

A. 1 and 2 B. 1 and 3 C. 2 and 3 D. 1 only E. all three

40. Let \vec{F} be any vector field whose components have continuous partial derivatives up to second order, let f be any real valued function with continuous partial derivatives up to second order, and let $\nabla = \vec{i} \frac{\partial}{\partial x} + \vec{j} \frac{\partial}{\partial y} + \vec{k} \frac{\partial}{\partial z}$. Find the incorrect statement.

- A. $\text{curl}(\text{grad } f) = \vec{0}$ B. $\text{div}(\text{curl } \vec{F}) = 0$ C. $\text{grad}(\text{div } \vec{F}) = 0$
 D. $\text{curl } \vec{F} = \nabla \times \vec{F}$ E. $\text{div } \vec{F} = \nabla \cdot \vec{F}$

41. A wire lies on the xy -plane along the curve $y = x^2$, $0 \leq x \leq 2$. The mass density (per unit length) at any point (x, y) of the wire is equal to x . The mass of the wire is

- A. $(17\sqrt{17} - 1)/12$ B. $(17\sqrt{17} - 1)/8$ C. $17\sqrt{17} - 1$
 D. $(\sqrt{17} - 1)/3$ E. $(\sqrt{17} - 1)/12$

42. Evaluate $\int_C \vec{F} \cdot d\vec{r}$ where $\vec{F}(x, y) = y\vec{i} + x^2\vec{j}$ and C is composed of the line segments from $(0, 0)$ to $(1, 0)$ and from $(1, 0)$ to $(1, 2)$.

- A. 0 B. $\frac{2}{3}$ C. $\frac{5}{6}$ D. 2 E. 3

43. Evaluate the line integral

$$\int_C x dx + y dy + xy dz$$

where C is parametrized by $\vec{r}(t) = \cos t\vec{i} + \sin t\vec{j} + \cos t\vec{k}$ for $-\frac{\pi}{2} \leq t \leq 0$.

- A. 1 B. -1 C. $\frac{1}{3}$ D. $-\frac{1}{3}$ E. 0

44. Are the following statements true or false?

1. The line integral $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy$ is independent of path in the xy -plane.
2. $\int_C (x^3 + 2xy)dx + (x^2 - y^2)dy = 0$ for every closed oriented curve C in the xy -plane.
3. There is a function $f(x, y)$ defined in the xy -plane, such that $\text{grad } f(x, y) = (x^3 + 2xy)\vec{i} + (x^2 - y^2)\vec{j}$.

- A. all three are false B. 1 and 2 are false, 3 is true C. 1 and 2 are true, 3 is false
 D. 1 is true, 2 and 3 are false E. all three are true

45. Evaluate $\int_C y^2 dx + 6xy dy$ where C is the boundary curve of the region bounded by $y = \sqrt{x}$, $y = 0$ and $x = 4$, in the counterclockwise direction.

- A. 0 B. 4 C. 8 D. 16 E. 32

46. If C goes along the x -axis from $(0, 0)$ to $(1, 0)$, then along $y = \sqrt{1 - x^2}$ to $(0, 1)$, and then back to $(0, 0)$ along the y -axis, then $\int_C xy \, dy =$

A. $-\int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx$

B. $\int_0^1 \int_0^{\sqrt{1-x^2}} y \, dy \, dx$

C. $-\int_0^1 \int_0^{\sqrt{1-x^2}} x \, dy \, dx$

D. $\int_0^1 \int_0^{\sqrt{1-x^2}} x \, dy \, dx$

E. 0

47. Evaluate $\int_C \vec{F} \cdot d\vec{r}$, if $\vec{F}(x, y) = (xy^2 - 1)\vec{i} + (x^2y - x)\vec{j}$ and C is the circle of radius 1 centered at $(1, 2)$ and oriented counterclockwise.

A. 2

B. π

C. 0

D. $-\pi$

E. -2

48. Green's theorem yields the following formula for the area of a simple region R in terms of a line integral over the boundary C of R , oriented counterclockwise. Area of $R = \iint_R dA =$

A. $-\int_C y \, dx$

B. $\int_C y \, dx$

C. $\int_C x \, dx$

D. $\frac{1}{2} \int_C y \, dx - x \, dy$

E. $-\int x \, dy$

49. Evaluate the surface integral $\iint_S x \, dS$ where S is the part of the plane $2x + y + z = 4$ in the first octant.

A. $8\sqrt{6}$

B. $\frac{8}{3}\sqrt{6}$

C. $\frac{8}{3}\sqrt{14}$

D. $\frac{\sqrt{14}}{3}$

E. $\frac{\sqrt{10}}{3}$

50. If S is the part of the paraboloid $z = x^2 + y^2$ with $z \leq 4$, \vec{n} is the unit normal vector on S directed upward, and $\vec{F}(x, y, z) = x\vec{i} + y\vec{j} + z\vec{k}$, then $\iint_S \vec{F} \cdot \vec{n} \, dS =$

A. 0

B. 8π

C. 4π

D. -4π

E. -8π

ANSWERS

1-C, 2-A, 3-B, 4-B, 5-A, 6-D, 7-B, 8-D, 9-E, 10-B, 11-A, 12-E, 13-B, 14-B,
15-D, 16-C, 17-E, 18-D, 19-A, 20-C, 21-E, 22-A, 23-D, 24-B, 25-E, 26-B, 27-C,
28-B, 29-E, 30-A, 31-B, 32-D, 33-B, 34-B, 35-A, 36-E, 37-B, 38-C, 39-B, 40-C,
41-A, 42-D, 43-D, 44-E, 45-D, 46-B, 47-D, 48-A, 49-B, 50-E, 51-A, 52-E