MA 266
FINAL EXAM INSTRUCTIONS
May 2, 2005

NAME ______________________ INSTRUCTOR ______________________

1. You must use a #2 pencil on the mark–sense sheet (answer sheet).

2. If the cover of your question booklet is GREEN, write 01 in the TEST/QUIZ NUMBER boxes and blacken in the appropriate spaces below. If the cover is ORANGE, write 02 in the TEST/QUIZ NUMBER boxes and darken the spaces below.

3. On the mark-sense sheet, fill in the instructor's name and the course number.

4. Fill in your NAME and OLD 9-DIGIT PURDUE ID NUMBER, (not your new PUID number) and blacken in the appropriate spaces.

5. Fill in the SECTION NUMBER boxes with the division and section number of your class. For example, for division 02, section 03, fill in 0203 and blacken the corresponding circles, including the circles for the zeros. (If you do not know your division and section number, ask your instructor.)

6. Sign the mark–sense sheet.

7. Fill in your name and your instructor's name on the question sheets above.

8. There are 25 questions, each worth 8 points. Blacken in your choice of the correct answer in the spaces provided for questions 1–25. Do all your work on the question sheets. Turn in both the mark–sense sheets and the question sheets when you are finished.

9. Show your work on the question sheets. Although no partial credit will be given, any disputes about grades or grading will be settled by examining your written work on the question sheets.

10. NO CALCULATORS, BOOKS, OR PAPERS ARE ALLOWED. Use the back of the test pages for scrap paper.

11. A table of Laplace Transforms can be found on the last page of the question sheets.

1. If $y(t) = \sin 2t$ is a solution of $y'' + 9y = f(t)$, then $f(t) =$

A. $\sin 2t$

B. $A\cos 3t$

C. 0

D. $\boxed{5\sin 2t}$

E. $13\sin 2t$

2. If $y = y(x)$ is the solution to

$$\frac{dy}{dx} = \frac{4xy}{2+x^2}, \qquad y(0) = 4,$$

then $y(\sqrt{2})$=

A. 4

B. $\boxed{16}$

C. 1

D. 2

E. $2\sqrt{2}$

3. The general solution to $x^2y' + 2xy = e^{5x}$ is

A. $y = \frac{1}{5}e^{5x} + c$

B. $\boxed{y = \frac{1}{5x^2}e^{5x} + cx^{-2}}$

C. $y = \frac{1}{5x^2}e^{5x}$

D. $y = ce^{5x}$

E. $y = \frac{5}{x^2}e^{5x} + c$

4. The solution to the problem

$$(2xy + x^3)dx + (x^2 + y^4)dy = 0, \quad y(0) = 1$$

is

A. $x + \frac{1}{5}y^5 = \frac{1}{5}$

B. $x^2y + 5y^5 = 5$

C. $\boxed{x^2y + \frac{x^4}{4} + \frac{y^5}{5} = \frac{1}{5}}$

D. $x^2y + 4x^4 + 5y^5 = 5$

E. $x^2y + x^4 + \frac{y^5}{5} = \frac{1}{5}$

5. The solution in implicit form of

$$\frac{dy}{dx} = \frac{x^2 + 3y^2}{2xy}$$

is:

A. $x^2 + y^2 = x^3 + C$

B. $\boxed{x^2 + y^2 = Cx^3}$

C. $x^2 + x^3 = y^2 + C$

D. $Cx^2 = x^3 + y^2$

E. $x^2 + y^3 + xy^2 = C$

6. Which of the following best describes the stability of equilibrium solutions for the autonomous differential equation $y' = y(4 - y^2)$?

A. $\boxed{y = 0 \text{ unstable; } y = 2 \text{ and } y = -2 \text{ both stable}}$

B. $y = 0$ unstable; $y = 2$ stable

C. $y = 0$ and $y = 2$ both stable

D. $y = 0$ stable; $y = 2$ unstable; $y = -2$ stable

E. $y = 0$ stable; $y = -2$ and $y = 2$ both unstable

7. Solve the initial value problem $y' - y = e^{-t}$ with $y(0) = a$. For what value(s) of a is the solution bounded (i.e., not tending to infinity as $t \to +\infty$) on the interval $t > 0$?

A. 2

B. $\boxed{-\frac{1}{2}}$

C. $-\frac{1}{3}$

D. all values

E. no values

8. Initially a tank holds 50 gallons of pure water. A salt solution containing $\frac{1}{3}$ lb of salt per gallon runs into the tank at the rate of 5 gallons per minute. The well mixed solution runs out of the tank at a rate of 2 gallons per minute. Let $x(t)$ be the amount of salt in the tank at time t. Find a differential equation satisfied by $x(t)$. (DO NOT SOLVE THE EQUATION)

A. $\boxed{\frac{dx}{dt} = \frac{5}{3} - \frac{2x}{50+3t}}$

B. $\frac{dx}{dt} = \frac{5}{2} - \frac{3x}{50+3t}$

C. $\frac{dx}{dt} = \frac{5}{3} - \frac{3x}{50+2t}$

D. $\frac{dx}{dt} = \frac{5}{2} - \frac{2x}{50+3t}$

E. $\frac{dx}{dt} = \frac{5}{3} - \frac{2x}{50+2t}$

9. The function $y_1 = t^2$ is a solution of the differential equation

$$t^2\frac{d^2y}{dt^2} - 2t\frac{dy}{dt} + 2y = 0.$$

Choose a function y_2 from the list below so that the pair y_1, y_2 form a fundamental set of solutions to the differential equation.

A. $y_2 = t^2 \sin t$

B. $y_2 = t^2 e^t$

C. $y_2 = t \sin t$

D. $\boxed{y_2 = t}$

E. $y_2 = t^2$

10. The largest open interval on which the solution to the initial value problem

$$\begin{cases} (\cos t)\, y' + \dfrac{t}{t-3}\, y = \ln(4-t) \\ \\ y(2) = 0 \end{cases}$$

is guaranteed by the Existence and Uniqueness Theorem to exist is

A. $-\frac{\pi}{2} < t < \frac{\pi}{2}$

B. $0 < t < \pi$

C. $\boxed{\frac{\pi}{2} < t < 3}$

D. $2 < t < 4$

E. $4 < t < \infty$

11. If $y(x)$ is the solution of $y'' - y' - 2y = 0$ satisfying $y(0) = 1$ and $y'(0) = -1$, then $y(1) =$

A. $\boxed{e^{-1}}$

B. e^2

C. $e^2 - e^{-1}$

D. $2e^{-1}$

E. $e^{-1} + 2e^2$

12. The general solution $y(t)$ of the differential equation

$$y'' - 3y' + 2y = 2e^t$$

is

A. $y(t) = c_1e^t + c_2e^{2t} + 2e^t$

B. $y(t) = c_1e^{-t} + c_2e^{2t} + e^t$

C. $y(t) = e^t + e^{2t} + cte^t$

D. $\boxed{y(t) = c_1e^t + c_2e^{2t} - 2te^t}$

E. $y(t) = c_1e^{-t} + c_2e^{-2t} + 2te^t$

13. The values of the constant r such that $y = x^r$ solves $x^2y'' + xy' - 2y = 0$ for $x > 0$ are

A. $1 \pm \sqrt{2}$

B. $\pm i\sqrt{2}$

C. $\boxed{\pm\sqrt{2}}$

D. $-1, -2$

E. $1, -2$

14. The proper form of the particular solution of the differential equation

$$y''' + 3y'' + 3y' + y = e^{-t}$$

used in the Method of Undetermined Coefficients is

A. Ae^{-t}

B. $A\cos t + B\sin t$

C. $At\cos t + Bt\sin t$

D. At^2e^{-t}

E. $\boxed{At^3e^{-t}}$

15. The inverse Laplace transform of $\frac{s^2+9s+2}{(s+3)(s-1)^2}$ is

A. $4e^t + 2te^t + e^{-3t}$

B. $2e^t + 3te^t - e^{3t}$

C. $2e^{-t} + 3te^{-t} + e^{3t}$

D. $2e^{-t} + 3te^{-t} + te^{-3t}$

E. $\boxed{2e^t + 3te^t - e^{-3t}}$

16. The Laplace transform of

$$f(t) = \int_0^t (t-\tau)e^{t-\tau} \cos\ 2\tau\ d\tau$$

is

A. $F(s) = \dfrac{1}{(s-1)(s^2+4)}$

B. $F(s) = \dfrac{s}{(s^2+1)(s^2+4)}$

C. $\boxed{F(s) = \dfrac{s}{(s-1)^2(s^2+4)}}$

D. $F(s) = \dfrac{2}{(s^2+1)(s+4)^2}$

E. $F(s) = \dfrac{s}{(s-4)^2(s-1)}$

17. Find the Laplace transform of

$$f(t) = \begin{cases} 2, & 0 \leq t < 2, \\ (t-2)^2, & t \geq 2. \end{cases}$$

A. $\frac{2}{s} + e^{-2s}(\frac{2}{s^3} - \frac{4}{s^2})$

B. $\frac{2}{s} + e^{-2s}(\frac{2}{s^3} - \frac{4}{s^2} + \frac{2}{s})$

C. $\frac{2}{s} + e^{-2s}(\frac{2}{s^3} + \frac{4}{s^2} + \frac{2}{s})$

D. $\frac{2}{s} + e^{-2s}\frac{2}{s^3}$

E. $\boxed{\frac{2}{s} + e^{-2s}(\frac{2}{s^3} - \frac{2}{s})}$

18. A mass weighing 16 lb stretches a spring $\frac{1}{2}$ ft. The mass is pulled down 1 ft from the equilibrium position, and then set in motion with a downward velocity of 8 ft/sec. Assuming that there is no air resistance and that the downward direction is the position direction. (The gravity constant $g = 32$ ft/sec^2.) Then, the amplitude of the oscillation is:

A. 1

B. $\boxed{\sqrt{2}}$

C. $-\sqrt{2}$

D. 2

E. $\frac{\pi}{4}$

19. The solution of the differential equation

$$y'' - 2y' + 2y = \delta(t-1), \quad y(0) = 0,\ y'(0) = 0$$

is

A. $\boxed{u_1(t)e^{t-1}\sin(t-1)}$

B. $\frac{e^{-s}}{s^2-2s+2}$

C. $\frac{u_1(t)}{t^2-2t+2}$

D. $u_1(t)e^t \sin t$

E. $u_1(t)e^{t+1}\sin(t+1)$

20. The general solution to

$$y^{(4)} + 2y'' + y = 0$$

is

A. $c_1 \cos t + c_2 \sin t$

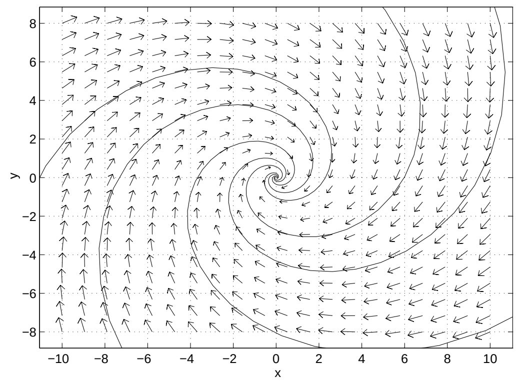
B. $c_1 e^t + c_2 e^{-t}$

C. $\boxed{c_1 \cos t + c_2 \sin t + c_3 t \cos t + c_4 t \sin t}$

D. $c_1 e^t \sin t + c_2 e^t \cos t$

E. $c_1 e^t + c_2 e^{-t} + c_3 \sin t + c_4 \cos t$

21. The phase portrait for a linear system of the form $\vec{x}\,' = \mathbf{A}\vec{x}$, where $\mathbf{A}$ is a 2×2 matrix is as follows.



If r_1 and r_2 denote the eigenvalues of $\mathbf{A}$, then what can you conclude about r_1 and r_2 by examining the phase portrait?

A. r_1 and r_2 are distinct and positive

B. r_1 and r_2 are distinct and negative

C. r_1 and r_2 have opposite signs

D. r_1 and r_2 are complex and have positive real part

E. r_1 and r_2 are complex and have negative real part

22. The function $x_2(t)$ determined by the initial value problem

$$\begin{aligned} x_1' &= x_2 \\ x_2' &= -x_1 \end{aligned}$$

with initial conditions $x_1(0) = 1$ and $x_2(0) = 1$ is given by

A. $x_2(t) = -\sin t + \cos t$

B. $x_2(t) = \sin t + \cos t$

C. $x_2(t) = \frac{1}{2}(e^t + e^{-t})$

D. $x_2(t) = \cos t$

E. $x_2(t) = ie^{it} - ie^{-it}$

23. Find the general solution of the first order system

$$\vec{x}\,' = \mathbf{A}\vec{x} \text{ where } \mathbf{A} = \begin{pmatrix} 6 & -4 \\ 1 & 2 \end{pmatrix}$$

given that $\vec{\xi} = \begin{pmatrix} 2 \\ 1 \end{pmatrix}$ is an eigenvector associated to the repeated eigenvalue $r = 4$ for the matrix $\mathbf{A}$, and that $\vec{\eta} = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ satisfies $(\mathbf{A} - 4\mathbf{I})\vec{\eta} = \vec{\xi}$.

A. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + c_2 t \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t}$

B. $\boxed{c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 2t+1 \\ t \end{pmatrix} e^{4t}}$

C. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{4t}$

D. $c_1 \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{4t} + c_2 \begin{pmatrix} t+2 \\ 1 \end{pmatrix} e^{4t}$

E. $c_1 \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{4t} + c_2 t \begin{pmatrix} 1 \\ 0 \end{pmatrix} e^{4t}$

24. Find the solution of the initial value problem

$$\vec{x}' = \begin{pmatrix} 1 & 1 \\ 4 & 1 \end{pmatrix} \vec{x} \text{ with } \vec{x}(0) = \begin{pmatrix} 2 \\ 0 \end{pmatrix}$$

A. $\boxed{\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{3t} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{-t}}$

B. $\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{3t} + \begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t}$

C. $-\frac{1}{2}\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{3t} + \frac{1}{2}\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{-t}$

D. $\begin{pmatrix} 1 \\ 2 \end{pmatrix} e^{-t} + \begin{pmatrix} 1 \\ -2 \end{pmatrix} e^{3t}$

E. $\frac{1}{2}\begin{pmatrix} 2 \\ 1 \end{pmatrix} e^{3t} - \frac{1}{2}\begin{pmatrix} -2 \\ 1 \end{pmatrix} e^{-t}$

25. Consider the system

$$\vec{x}' = \begin{pmatrix} \alpha & 1 \\ 1 & \alpha \end{pmatrix} \vec{x}$$

For what values of α is the equilibrium solution $\vec{x} = 0$ an asymptotically stable node?

A. no value of α

B. $\boxed{\alpha < -1}$

C. $\alpha > 1$

D. $-1 < \alpha < 1$

E. all real α

	$f(t) = \mathcal{L}^{-1}\{F(s)\}$	$F(s) = \mathcal{L}\{f(t)\}$
1.	1	$\dfrac{1}{s}$
2.	e^{at}	$\dfrac{1}{s-a}$
3.	t^n	$\dfrac{n!}{s^{n+1}}$
4.	$t^p \ (p > -1)$	$\dfrac{\Gamma(p+1)}{s^{p+1}}$
5.	$\sin at$	$\dfrac{a}{s^2+a^2}$
6.	$\cos at$	$\dfrac{s}{s^2+a^2}$
7.	$\sinh at$	$\dfrac{a}{s^2-a^2}$
8.	$\cosh at$	$\dfrac{s}{s^2-a^2}$
9.	$e^{at}\sin bt$	$\dfrac{b}{(s-a)^2+b^2}$
10.	$e^{at}\cos bt$	$\dfrac{s-a}{(s-a)^2+b^2}$
11.	$t^n e^{at}$	$\dfrac{n!}{(s-a)^{n+1}}$
12.	$u_c(t)$	$\dfrac{e^{-cs}}{s}$
13.	$u_c(t)f(t-c)$	$e^{-cs}F(s)$
14.	$e^{ct}f(t)$	$F(s-c)$
15.	$f(ct)$	$\dfrac{1}{c}F\left(\dfrac{s}{c}\right), \quad c > 0$
16.	$\displaystyle\int_0^t f(t-\tau)\,g(\tau)\,d\tau$	$F(s)\,G(s)$
17.	$\delta(t-c)$	e^{-cs}
18.	$f^{(n)}(t)$	$s^nF(s) - s^{n-1}f(0) - \cdots - sf^{(n-2)}(0) - f^{(n-1)}(0)$
19.	$(-t)^n f(t)$	$F^{(n)}(s)$