Computer Project # 1

Nonlinear Springs

**Goal:** Investigate the behavior of nonlinear springs.

**Tools needed:** ode45, plot routines.

**Description:** Sometimes for certain (nonlinear) spring-mass systems, the spring force is not given by Hooke’s Law but instead satisfies

$$F_{\text{Spring}} = k u + \epsilon u^3$$

where $k > 0$ is the spring constant and $\epsilon$ is small but may be positive or negative and represents the “strength” of the spring ($\epsilon = 0$ gives Hooke’s Law). The spring is called a **hardening spring** if $\epsilon > 0$ and a **softening spring** if $\epsilon < 0$.

![Diagram of a spring-mass system]

**Questions:** Suppose a nonlinear spring-mass system satisfies the initial value problem

\[
\begin{align*}
&u'' + u + \epsilon u^3 = 0 \\
&u(0) = 0, \ u'(0) = 1
\end{align*}
\]

(1) Let $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ and plot the solutions of the above initial value problem for $0 \leq t \leq 15$. Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as $\epsilon$ increases? Let $T_1 =$ first time the mass reaches equilibrium after $t = 0$. Estimate $T_1$ when $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. What appears to happen to $T_1$ as $\epsilon$ increases?

(2) Let $\epsilon = -0.1, -0.2, -0.3, -0.4$ and plot the solutions of the above initial value problem for $0 \leq t \leq 15$. Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as $\epsilon$ decreases? Let $T_1 =$ first time the mass reaches equilibrium after $t = 0$. Estimate $T_1$ when $\epsilon = -0.1, -0.2, -0.3, -0.4$. What appears to happen to $T_1$ as $\epsilon$ decreases?