MA 224 FORMULAS

THE SECOND DERIVATIVE TEST

Suppose f is a function of two variables x and y, and that all the second-order partial derivatives are continuous. Let

$$D = f_{xx}f_{yy} - (f_{xy})^2$$

and suppose (a, b) is a critical point of f.

1. If $D(a, b) < 0$, then f has a saddle point at (a, b),
2. If $D(a, b) > 0$ and $f_{xx}(a, b) < 0$, then f has a relative maximum at (a, b).
3. If $D(a, b) > 0$ and $f_{xx}(a, b) > 0$, then f has a relative minimum at (a, b).
4. If $D(a, b) = 0$, the test is inconclusive.

LEAST-SQUARES LINE

The equation of the least-squares line for the n points (x_1,y_1), (x_2,y_2), ... , (x_n,y_n), is $y = mx + b$, where m and b are solutions to the system of equations

$$\begin{aligned} (x_1^2 + x_2^2 + \cdots + x_n^2)m + (x_1 + x_2 + \cdots + x_n)b &= x_1y_1 + x_2y_2 + \cdots + x_ny_n \\ (x_1 + x_2 + \cdots + x_n)m + nb &= y_1 + y_2 + \cdots + y_n \end{aligned}$$

TRAPEZOIDAL RULE

$$\int_a^b f(x)dx \equiv \frac{\Delta x}{2}\Big[f(x_0) + 2f(x_1) + 2f(x_2) + \cdots + 2f(x_{n-1}) + f(x_n)\Big],$$

where $a = x_0, x_1, x_2, \ldots, x_n = b$ subdivides $[a, b]$ into n equal subintervals of length $\Delta x = \dfrac{b-a}{n}$.

ERROR ESTIMATE FOR THE TRAPEZOIDAL RULE

If M is the maximum value of $|f''(x)|$ on the interval $a \le x \le b$, then

$$|E_n| \le \frac{M(b-a)^3}{12n^2}$$

GEOMETRIC SERIES

If $0 < |r| < 1$, then

$$\sum_{n=0}^{\infty} ar^n = \frac{a}{1-r}$$

TAYLOR SERIES

The Taylor series of $f(x)$ about $x = a$ is the power series

$$\sum_{n=0}^{\infty} \frac{f^{(n)}(a)}{n!}(x-a)^n = f(a) + f'(a)(x-a) + \frac{f^{(2)}(a)}{2!}(x-a)^2 + \ldots$$

Examples: (with $a = 0$)

$$e^x = \sum_{n=0}^{\infty} \frac{x^n}{n!}, \text{ for } -\infty < x < \infty; \qquad \ln x = \sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n}(x-1)^n, \text{ for } 0 < x \le 2$$