

Table of Integrals

Forms Involving $a + bu$

1. $\int \frac{u \, du}{a + bu} = \frac{1}{b^2} [a + bu - a \ln|a + bu|] + C$
2. $\int \frac{u^2 \, du}{a + bu} = \frac{1}{2b^3} [(a + bu)^2 - 4a(a + bu) + 2a^2 \ln|a + bu|] + C$
3. $\int \frac{u \, du}{(a + bu)^2} = \frac{1}{b^2} \left[\frac{a}{a + bu} + \ln|a + bu| \right] + C$
4. $\int u \sqrt{a + bu} \, du = \frac{2}{15b^2} (3bu - 2a)(a + bu)^{3/2} + C$
5. $\int \frac{u \, du}{\sqrt{a + bu}} = \frac{2}{3b^2} (bu - 2a)\sqrt{a + bu} + C$
6. $\int \frac{du}{u \sqrt{a + bu}} = \frac{1}{\sqrt{a}} \ln \left| \frac{\sqrt{a + bu} - \sqrt{a}}{\sqrt{a + bu} + \sqrt{a}} \right| + C \quad (\text{if } a > 0)$

Forms Involving $\sqrt{a^2 + u^2}$

7. $\int \sqrt{a^2 + u^2} \, du = \frac{u}{2} \sqrt{a^2 + u^2} + \frac{a^2}{2} \ln|u + \sqrt{a^2 + u^2}| + C$
8. $\int u^2 \sqrt{a^2 + u^2} \, du = \frac{u}{8} (a^2 + 2u^2) \sqrt{a^2 + u^2} - \frac{a^4}{8} \ln|u + \sqrt{a^2 + u^2}| + C$
9. $\int \frac{du}{\sqrt{a^2 + u^2}} = \ln|u + \sqrt{a^2 + u^2}| + C$
10. $\int \frac{du}{u \sqrt{a^2 + u^2}} = -\frac{1}{a} \ln \left| \frac{\sqrt{a^2 + u^2} + a}{u} \right| + C$
11. $\int \frac{du}{u^2 \sqrt{a^2 + u^2}} = -\frac{\sqrt{a^2 + u^2}}{a^2 u} + C$
12. $\int \frac{du}{(a^2 + u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 + u^2}} + C$

Forms Involving $\sqrt{u^2 - a^2}$

13. $\int \sqrt{u^2 - a^2} \, du = \frac{u}{2} \sqrt{u^2 - a^2} - \frac{a^2}{2} \ln|u + \sqrt{u^2 - a^2}| + C$
14. $\int u^2 \sqrt{u^2 - a^2} \, du = \frac{u}{8} (2u^2 - a^2) \sqrt{u^2 - a^2} - \frac{a^4}{8} \ln|u + \sqrt{u^2 - a^2}| + C$
15. $\int \frac{\sqrt{u^2 - a^2}}{u^2} \, du = -\frac{\sqrt{u^2 - a^2}}{u} + \ln|u + \sqrt{u^2 - a^2}| + C$
16. $\int \frac{du}{\sqrt{u^2 - a^2}} = \ln|u + \sqrt{u^2 - a^2}| + C$

$$17. \int \frac{du}{u^2 \sqrt{u^2 - a^2}} = \frac{\sqrt{u^2 - a^2}}{a^2 u} + C$$

$$18. \int \frac{du}{(u^2 - a^2)^{3/2}} = -\frac{u}{a^2 \sqrt{u^2 - a^2}} + C$$

Forms Involving $\sqrt{a^2 - u^2}$

$$19. \int \frac{\sqrt{a^2 - u^2}}{u} du = \sqrt{a^2 - u^2} - a \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$20. \int \frac{du}{u \sqrt{a^2 - u^2}} = -\frac{1}{a} \ln \left| \frac{a + \sqrt{a^2 - u^2}}{u} \right| + C$$

$$21. \int \frac{du}{u^2 \sqrt{a^2 - u^2}} = -\frac{\sqrt{a^2 - u^2}}{a^2 u} + C$$

$$22. \int \frac{du}{(a^2 - u^2)^{3/2}} = \frac{u}{a^2 \sqrt{a^2 - u^2}} + C$$

Forms Involving e^{au} and $\ln u$

$$23. \int u e^{au} du = \frac{1}{a^2} (au - 1) e^{au} + C$$

$$24. \int u^n e^{au} du = \frac{1}{a} u^n e^{au} - \frac{n}{a} \int u^{n-1} e^{au} du$$

$$25. \int \frac{du}{1 + be^{au}} = u - \frac{1}{a} \ln(1 + be^{au}) + C$$

$$26. \int \ln u du = u \ln u - u + C$$

$$27. \int u^n \ln u du = \frac{u^{n+1}}{(n+1)^2} [(n+1) \ln u - 1] + C \quad (n \neq -1)$$

$$28. \int \frac{du}{u \ln u} = \ln |\ln u| + C$$

$$29. \int (\ln u)^n du = u (\ln u)^n - n \int (\ln u)^{n-1} du$$
