Solution for HWs (§1.1)

#1.2.1 (a) Proof. Assume that $\sqrt{3}$ is rational

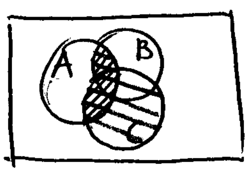
$\Rightarrow \exists\, p, q \in \mathbb{Z}$, that do not have common factor, s.t. $\left(\frac{p}{q}\right)^2 = 3$

$\Rightarrow p^2 = 3q^2 \Rightarrow p = 3k$ for some $k \in \mathbb{Z}$ (this can be proved by contradiction by letting $p = 3k+1$ and $p = 3k+2$)

$\Rightarrow (3k)^2 = 3q^2 \Rightarrow q^2 = 3k^2 \Rightarrow q = 3K$ for some $K \in \mathbb{Z}$.

$\Rightarrow$ p and q have a common factor 3, which is a contradiction.

#1.2.2 (a) false. $[0,1] \supset [0,\frac{1}{2}] \supset \cdots \supset [0,\frac{1}{n}] \supset \cdots$, but $\bigcap_{n=1}^{\infty} A_n = \{0\}$.

(c) 

$A = B = \{1, 2\}$, $C = \{1, \cancel{2}, 3\}$

$A \cap (B \cup C) = \{1, 2\}$, $(A \cap B) \cup C = \{1, 2, 3\}$

#1.2.3 (a) $x \in (A \cap B)^c \Rightarrow x \notin A \cap B$

$\Rightarrow x \notin A$ or $x \notin B$ $\quad (x \in A \cap B \Leftrightarrow x \in A$ and $x \in B)$

$\Rightarrow x \in A^c$ or $x \in B^c \Rightarrow x \in A^c \cup B^c$

or

Assume that $x \notin A^c \cup B^c$, then $x \notin A^c$ and $x \notin B^c$

$\Rightarrow x \in A$ and $x \in B \Rightarrow x \in A \cap B$ which is contradicted to $x \notin A \cap B$.

(b) $\forall x \in A^c \cup B^c \Rightarrow x \in A^c$ or $x \in B^c \Rightarrow x \notin A$ or $x \notin B$

$\Rightarrow x \notin A \cap B \Rightarrow x \in (A \cap B)^c$.