Solution for HWs (§1.2)

#1.2.5 (a) $|a-b| = |a+(-b)| \le |a| + |-b| = |a| + |b|$

(b) $||a|-|b|| \le |a-b| \Leftrightarrow -|a-b| \le |a|-|b| \le |a-b|$

$$\Leftrightarrow \begin{cases} |b| \le |a| + |a-b| & \Leftarrow \quad |b| = |(b-a)+a| \le |a| + |a-b| \\ |a| \le |b| + |a-b| & \end{cases}$$

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#1.2.6 (b) $A=[0,1]$, $B=[1,2]$, $f(x) = \begin{cases} 1, & x \in Q \\ 0, & x \notin Q \end{cases}$

$f(A\cap B) = \{1\}$, $f(A)\cap f(B) = \{0,1\}$

(c) $\forall y \in g(A\cap B) \Rightarrow \exists\ x \in A\cap B$ s.t. $y = g(x)$

$\Rightarrow x\in A$ and $x \in B$, $y = g(x) \Rightarrow y \in g(A)$ and $y \in g(B)$

$\Rightarrow y \in g(A) \cap g(B)$ #

(d) $g(A\cup B) = g(A) \cup g(B)$

$\forall y \in g(A\cup B) \Leftrightarrow \exists x \in A\cup B$ s.t. $y = g(x) \Leftrightarrow y = g(x)$ where $x\in A$ or $x \in B$

$\Leftrightarrow y \in g(A)$ or $y \in g(B) \Longleftrightarrow y \in g(A) \cup g(B)$ #

#1.2.10 (a) $y_1 = 1 < 4$; Assume that $y_k < 4$, want to prove $y_{k+1} < 4$.

$$y_{k+1} = \frac{3y_k + 4}{4} = \frac{3}{4} y_k + 1 < \frac{3}{4}\cdot 4 + 1 = 4.$$

(b) $y_2 = \frac{3}{4}y_1 + 1 = \frac{3}{4} + 1 > 1 = y_1$; Assume that $y_k > y_{k-1}$, want to prove $y_{k+1} > y_k$.

$$\begin{cases} y_{k+1} = \frac{3}{4} y_k + 1 \\ y_k = \frac{3}{4} y_{k-1} + 1 \end{cases} \Rightarrow (y_{k+1} - y_k) = \frac{3}{4}(y_k - y_{k-1}) > 0 \Rightarrow y_{k+1} > y_k.$$ #