Solution for HWs (§2.2)

#2.2.1 (a) $\dfrac{1}{6n^2+1} < \dfrac{1}{6n^2} < \varepsilon \Rightarrow n > \sqrt{\dfrac{1}{6\varepsilon}}$

(b) $\left|\dfrac{3n+1}{2n+5} - \dfrac{3}{2}\right| = \left|\dfrac{2(3n+1) - 3(2n+5)}{2(2n+5)}\right| = \dfrac{13}{2}\cdot\dfrac{1}{2n+5} < \dfrac{13}{4n} < \varepsilon \Rightarrow n > \dfrac{13}{4\varepsilon}$

(c) $\dfrac{2}{\sqrt{n+3}} < \dfrac{2}{\sqrt{n}} < \varepsilon \Rightarrow n > \left(\dfrac{2}{\varepsilon}\right)^2.$

#2.2.2 it implies the sequence is bounded

$$|x_n| \leq |x_n - x| + |x| < \varepsilon + |x|.$$

$$\{x_n\} = \{1, 0, 1, 0, \cdots\}$$

#2.2.4 For $\varepsilon \leq 1$, e.g., $\varepsilon = \frac{1}{2}$, for any N, $\exists\, n > N$ s.t. $|x_n| = 1 > \varepsilon = \frac{1}{2}$.

$\Rightarrow$ it diverges

For $\varepsilon \leq 1$, there is no response N.

For $\varepsilon > 1$, $|x_n| \leq 1 < \varepsilon$, for $n = 1, 2, \cdots$.

#2.2.7 Def $\lim\limits_{n\to\infty} x_n = \infty \Leftrightarrow \forall M > 0, \exists N$, s.t., $x_n > M$ for $n > N$.

Proof $\forall M > 0$, $\sqrt{n} > M \Leftrightarrow n > M^2 = N$.

(b) $\{1, 0, 2, 0, 3, 0, \cdots\}$ does not converges to $+\infty$.