Computer Project # 1 Nonlinear Springs

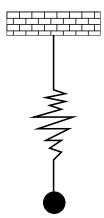
<u>Goal</u>: Investigate the behavior of nonlinear springs.

Tools needed: ode45, plot routines.

Description: Sometimes for certain (nonlinear) spring-mass systems, the spring force is not given by Hooke's Law but instead satisfies

$$F_{\text{Spring}} = k \, u + \epsilon \, u^3$$

where k > 0 is the spring constant and ϵ is small but may be positive or negative and represents the "strength" of the spring ($\epsilon = 0$ gives Hooke's Law). The spring is called a <u>hardening spring</u> if $\epsilon > 0$ and a softening spring if $\epsilon < 0$.



Questions: Suppose a nonlinear spring-mass system satisfies the initial value problem

$$\left\{ \begin{array}{l} u'' + u + \epsilon \, u^3 = 0 \\ u(0) = 0, \ u'(0) = 1 \end{array} \right. .$$

- (1) Let $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$ and plot the solutions of the above initial value problem for $0 \le t \le 15$. Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as ϵ increases? Let $T_1 =$ first time the mass reaches equilibrium after t = 0. Estimate T_1 when $\epsilon = 0.0, 0.2, 0.4, 0.6, 0.8, 1.0$. What appears to happen to T_1 as ϵ increases?
- (2) Let $\epsilon = -0.1, -0.2, -0.3, -0.4$ and plot the solutions of the above initial value problem for $0 \le t \le 15$. Estimate the maximum amplitude of the spring. What appears to happen to the amplitude as ϵ decreases ? Let $T_1 =$ first time the mass reaches equilibrium after t = 0. Estimate T_1 when $\epsilon = -0.1, -0.2, -0.3, -0.4$. What appears to happen to T_1 as ϵ decreases ?