Computer Project # 2

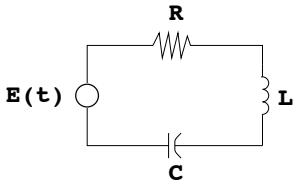
RLC-Circuits

Goal: Investigate the charge on a capacitor in an RLC circuit with varying voltage.

Tools needed: ode45, plot routines.

Description: If $Q(t) =$ charge on a capacitor at time t in an RLC circuit (with R, L and C being the resistance, inductance and capacitance, respectively, and $E(t) =$ applied voltage), then Kirchoff's Laws give the following 2^{nd} order differential equation for $Q(t)$:

$$(*) \qquad L\,Q''(t) + R\,Q'(t) + \frac{1}{C}\,Q(t) = E(t)$$



Questions: Assume $L = 1$, $C = \frac{1}{5}$, $R = 4$ and $E(t) = 8\cos\omega t$.

(1) Use **ode45** (and plot routines) to plot the solution of $(*)$ with $Q(0) = 0$ and $Q'(0) = 0$ over the interval $0 \le t \le 50$, for these values of ω : $\omega = 0.0,\ 1.0,\ 2.0,\ 4.0,\ 10.0,\ 20.0$.

(2) For each of these 6 plots, find the largest value of $|Q(t)|$ over $30 \le t \le 50$ and fill in the table:

ω	Max value of $\lvert Q(t)\rvert$ on $30 \le t \le 50$
0.0	
1.0	
2.0	
4.0	
10.0	
20.0	

(3) Does increasing ω appear to increase the maximum charge $|Q(t)|$ on the capacitor ? Interpret this result in terms of an equivalent spring-mass system.

Remark: There is an analogy between spring-mass systems and RLC circuits given by :

Spring-Mass System	RLC Circuit
$\boldsymbol{mx'' + cx' + kx = F(t)}$	$\boldsymbol{LQ'' + RQ' + \frac{1}{C}Q = E(t)}$
$x =$ Displacement	$Q =$ Charge
$x' =$ Velocity	$Q' = I =$ Current
$m =$ Mass	$L =$ Inductance
$c =$ Damping constant	$R =$ Resistance
$k =$ Spring constant	$1/C =$ (Capacitance)$^{-1}$
$F(t) =$ External force	$E(t) =$ Voltage